Hybrid Approaches for 0-1 Mixed Integer Program

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Ecole Polytechnique

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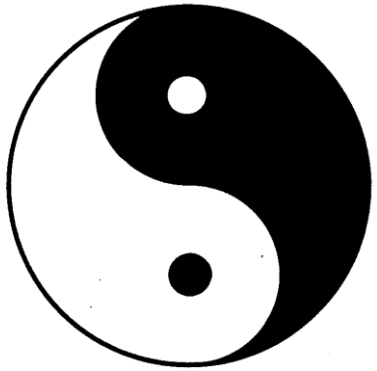
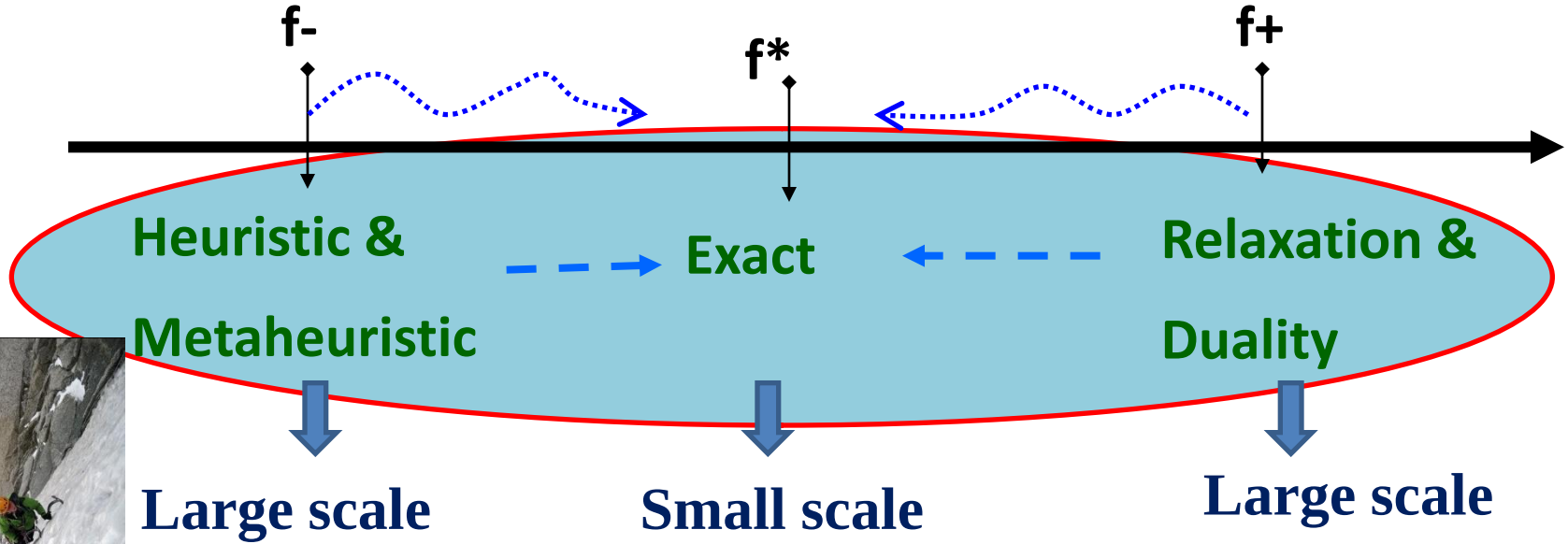


Hybrid Approaches

- **Hard Optimization Problem**

$$f^* = \text{Max}\{f(x) : x \in X \subseteq E\}$$



Google ROADEF/EURO challenge 2011-2012
Team S21

Hybrid Method for Machine Reassignment Problem

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Size of MIP

	m	*p*	#0-1var	#var	#cont
b_1	100	5 000	525 120	1 201	300 504
b_2	100	5 000	524 620	1 301	294 354
b_3	100	20 000	2 150 250	601	1 778 725
b_4	500	20 000	10 086 600	3 501	988 224
b_5	100	40 000	4 350 820	601	4 145 394
b_6	200	40 000	8 734 000	1 401	3 814 160
b_7	4 000	40 000	160 752 500	28 001	61 125 850
b_8	100	50 000	5 450 300	301	5 318 910
b_9	1 000	50 000	50 460 900	4 001	5 156 163
b_10	5 000	50 000	250 489 600	20 001	25 073 872

An example

Instance	Lower Bound	Upper Bound
A2-1	65	391 189 190

Some Contributions with Hybridization

Exact Methods

Branch-and-Bound

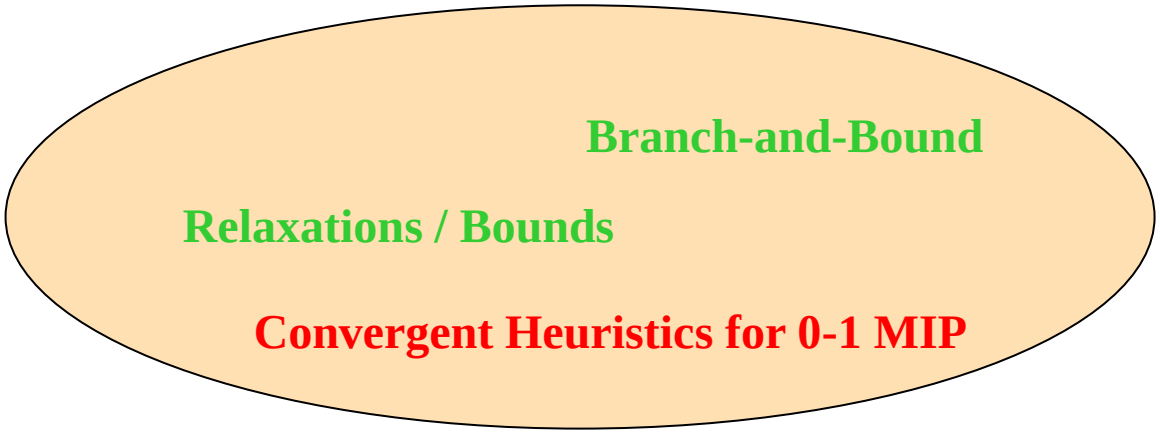
Relaxations / Bounds

Dynamic Programming

Heuristics / Metaheuristics

Scatter Search

Tabu Search
Oscillation Strategy
Variable Neighbourhood Se



Other techniques

- **Decomposition / Projection**
- **Fixation: Temporary or definitvely**
- **Cuts / Pseudo-Cuts**

Adaptive Memory
Intensification & Diversification

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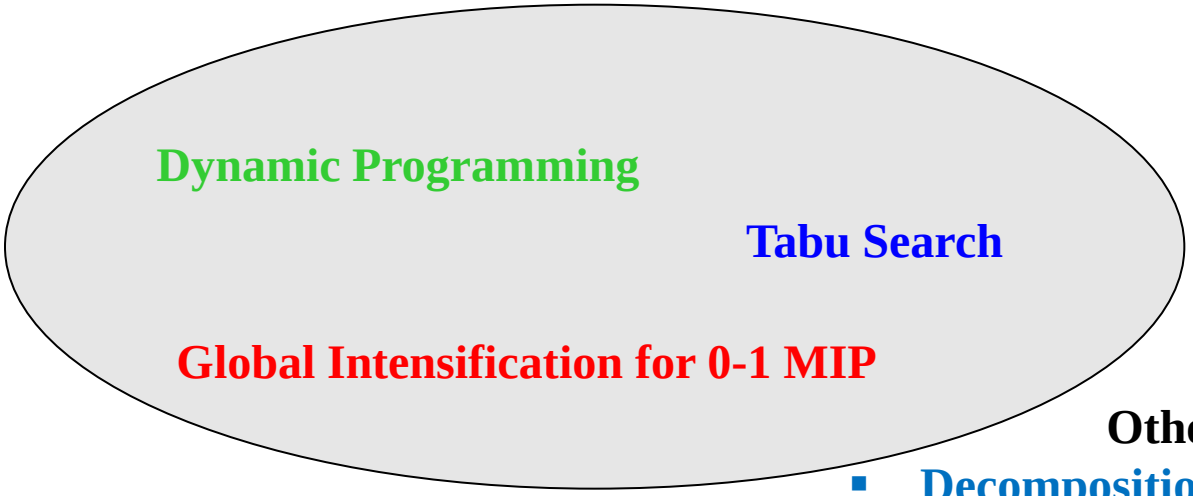

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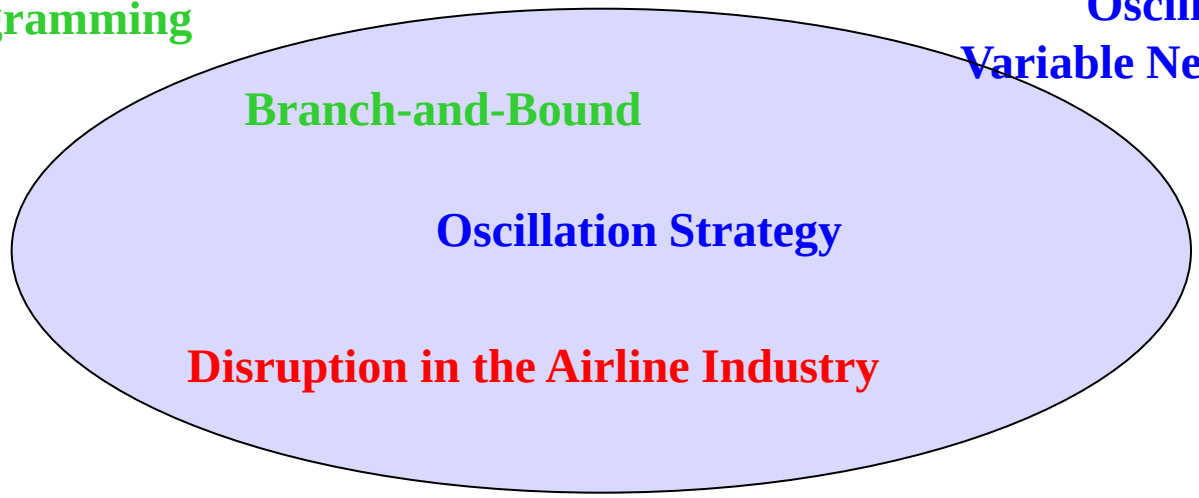

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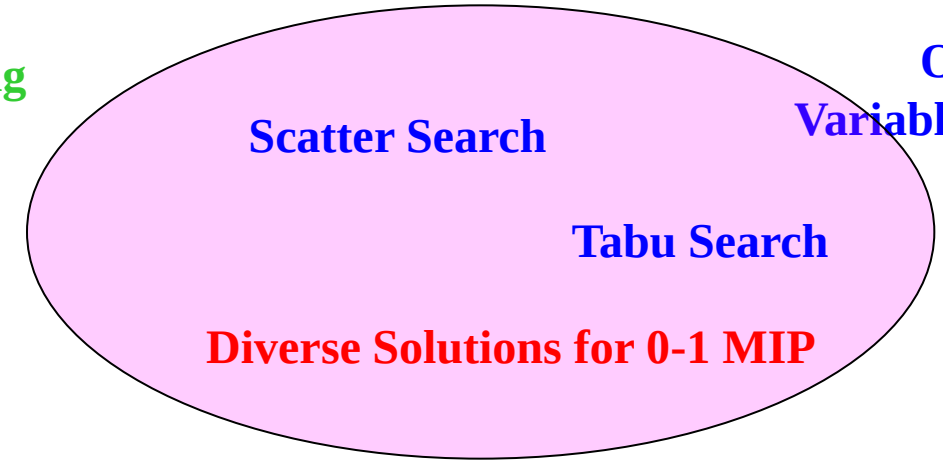

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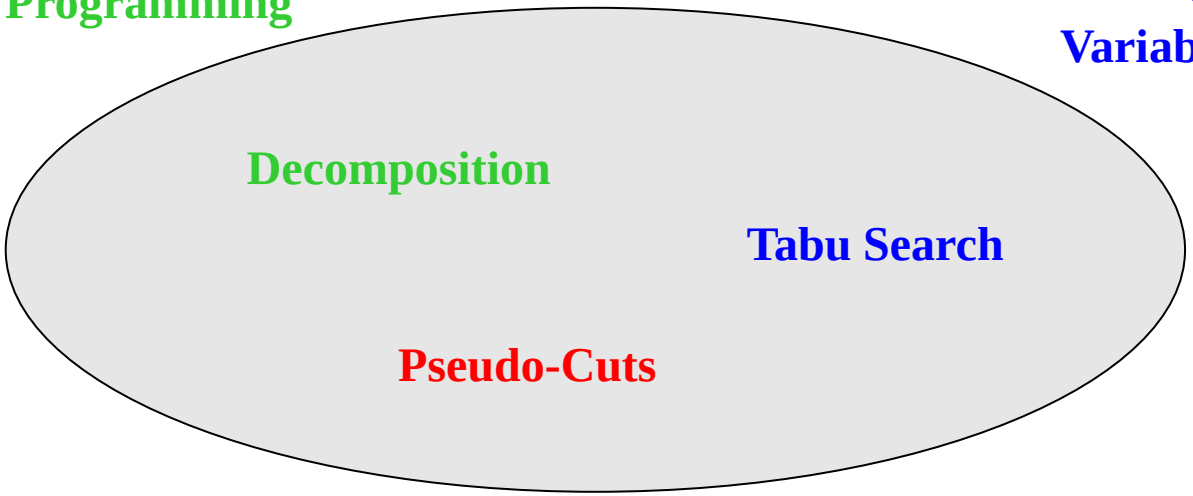

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Adaptive Memory

Intensification & Diversification

Outline

- MIP & Variants of Knapsack problem
- Relaxations of MIP
- Iterative Relaxation-based Heuristics (IRH)
- Hybrid VNDS with IRH
- Tabu Search & Dynamic Programming
- Tabu Search with Relaxation & Decomposition
- Conclusions

Mixed Integer Program

$$(MIP)\begin{cases} \texttt{maximiser} & cx \\ \texttt{s.c.} & Ax \leq b \\ & 0 \leq x_j \leq u_j \quad j = 1, \ldots, n \\ & x_j \ \texttt{entier} \quad j = 1, \ldots, p \end{cases}$$

n : nombre de variables
p : nombre de variables discrètes
m : nombre de contraintes

c(*n*), *A*(*m*,*n*), *b*(*m*)

- Programmation linéaire **(*PL*)** **$p = 0$**
- Progr. en variables entières **(*IP*)** **$p = n$**
- Progr. en var. 0-1 mixtes **(0-1*MIP*)** **$u_j = 1 \quad j = 1, \ldots, p$**
- Progr. en var. 0-1 **(0-1*IP*)** **$u_j = 1 \quad j = 1, \ldots, p \quad n = p$**
- Sac-à-dos multidimensionnel **(*SADM*)** **$u_j = 1 \quad j = 1, \ldots, p \quad n = p$**

$A, b, c \geq 0$

Multidimensional Knapsack Problem

$$(MKP) \quad \begin{cases} \max \ cx \\ s.t. \ Ax \le b \\ x \in \{0,1\}^n \end{cases} \qquad A(m,n), b(m), c(n) \ge 0$$

MKP* is *NP-Hard

- **Dantzig 1957, Lorie & Savage 1955, Markowitz & Manne 1957**
- **Martello & Toth, 1990; Kellerer & Pferschy & Pisinger 2004**
- **Fréville & Hanafi 2005, Hanafi & Wilbaut 2009**
- **Boussier & Vasquez & Vimont & Hanafi & Michelon 2009**
- **Hanafi & Lazic & Mladenovic & Wilbaut 2009**

Applications

- **Resource allocation problems**
- **Production scheduling problems**
- **Transportation management problems**
- **Adaptive multimedia system with QoS**
- **Telecommunication networks problems**
- **Wireless switch design**
- **Service selection for web services with QoS constraints**
- **Polyedral approach, (1,*k*)-configuration**

- **Sub-problem : Lagrangean / Surrogate relaxation**
- **Benchmark**

Variant of Knapsack Problem

- **Unidimensional Knapsack Problem** → $m = 1$
- **Bi-Dimensional Knapsack Problem** → $m = 2$

Objective

- *Quadratic*
- *Hyperbolic*
- *Min-Max*
- *Multi-Objectives*
- *Bi-Level*

Constraints

- *Cardinality*
- *Multiple-choice*
- *Precedence*
- *Demand*
- *Quadratic*

Multidemand Multidimensional Knapsack Problem

Cappanera & Trubian 2004

Knapsack & Covering Problem

$$(MDMKP)\begin{cases} \texttt{maximize} & \sum_{j=1}^{n} c_j x_j \\ \texttt{subject to} & \sum_{i=1}^{n} a_{1j}^{i} x_j \leq b_{1i} \quad \forall i = 1, \ldots, m_1 \\ & \sum_{i=1}^{n} a_{2j}^{i} x_j \geq b_{2i} \quad \forall i = 1, \ldots, m_2 \\ & x_j \in \{0;1\} \qquad j = 1, \ldots, n \end{cases}$$

MDMKP is a generalization of MKP

Multi-Choice Multidimensional Knapsack Problem

Nauss 78, Sinha & Zoltners 79

$$(MMKP)\quad \begin{cases} \max & \displaystyle\sum_{i=1}^{g}\sum_{j=1}^{g_i} c_{ij}x_{ij} & \\ s.t. & \displaystyle\sum_{i=1}^{g}\sum_{j=1}^{g_i} a_{ij}^{k}x_{ij} \le b^{k} & k=1,\ldots,m \\ & \displaystyle\sum_{j=1}^{g_i} x_{ij} = 1 & i=1,\ldots,g \\ & x_{ij} \in \{0,1\} & i=1,\ldots,g,\ j=1,\ldots,g_i \end{cases}$$

- g : **Number of groups**
- g_i : **Number of items in group i**
- c_{ij} : **Profit associated to item j of group i**
- a_{ij}^{k} : **Consuming resource k by item j of group i**
- b^{k} : **Capacity of resource k**

Multidimensional Knapsack Problem with Generalized Upper Bound constraints

$$(GUBMKP)\quad \begin{cases} \max & \displaystyle\sum_{i=1}^{g}\sum_{j=1}^{g_i} c_{ij}x_{ij} & \\ s.t. & \displaystyle\sum_{i=1}^{g}\sum_{j=1}^{g_i} a_{ij}^{k}x_{ij} \le b^{k} & k=1,\ldots,m \\ & \displaystyle\sum_{j=1}^{g_i} x_{ij} \le 1 & i=1,\ldots,g \\ & x_{ij} \in \{0,1\} & i=1,\ldots,g,\ j=1,\ldots,g_i \end{cases}$$

GUBMKP is a relaxation of MMKP

Knapsack Constrained Maximum Spanning Tree Problem

Aggarwal et al (1982)

$$(KCMST)\quad \begin{cases} \max & \sum_{a\in A} c_a x_a \\ s.t. & \sum_{a\in A} w_a x_a \le b \\ & x \text{ covering tree of } G \\ & x_a \in \{0,1\} \qquad a \in A \end{cases}$$

KCMST problem arises in practice in situations where the aim is to design a mobile communication network under a strict limit on total costs

Generalized Knapsack Problem

Suzuki (1978)

$$(GKP)\quad \begin{cases} \max & \sum_{j=1}^{n} c_j x_j \\ s.t. & \sum_{j=1}^{n} x_j \le c \\ & a_j y_j \le x_j \le b_j y_j \qquad j = 1,\ldots,n \\ & x_j \ge 0; y_j \in \{0,1\} \qquad j = 1,\ldots,n \end{cases}$$

Disjunctively Constrained Knapsack Problem

Yamada et al (2002)

$$(DCKP)\begin{cases} \max & \sum_{j=1}^{n} c_j x_j \\ s.t. & \sum_{j=1}^{n} a_j x_j \le b \\ & x_i + x_j \le 1 \qquad (i,j) \in I \\ & x_j \in \{0,1\} \qquad j = 1,\ldots,n \end{cases}$$

Max Min Multidimensional Knapsack Problem

$$(MMMKP)\begin{cases} Max \;\; Min\left\{c^k x : k = 1,\ldots, p\right\} \\ \quad s.t. \\ \quad\quad Ax \le b \quad x \in \{0,1\}^n \end{cases}$$

Introduction to minimax, Demyanov Molozemov 1974

Minimax and applications, Du, Pardalos 1995

Robust optimization applications, Yu 1996

A virtual pegging approach, Taniguchi, Yamada, Kataoka 2009

Iterative Heuristics, Hanafi, Mansi, Wilbaut 2012

Knapsack Sharing Problem

Brown (1979)

$$(KSP)\quad \begin{cases} \max \quad \min\left\{\sum_{j\in N_i} c_j x_j : i = 1,\ldots,m\right\} & \\ s.t. \quad \sum_{j\in N} a_j x_j \le b & \\ N = \bigcup_{i=1}^{m} N_i, N_i \cap N_l = \Phi & i \ne l \\ x_j \in \{0,1\} & j \in N \end{cases}$$

KSP is a generalization of KP

Knapsack Problem with Setup

Ham et al. (1985)

$$(KPS)\begin{cases} Max \ \sum_{i=1}^{N}\sum_{j=1}^{n_i} c_{ij}x_{ij} + \sum_{i=1}^{N} f_i y_i \\ \text{s.t} \quad \sum_{i=1}^{N}\sum_{j=1}^{n_i} a_{ij}x_{ij} + \sum_{i=1}^{N} d_i y_i \leq b \\ x_{ij} \leq y_i, j = 1, \dots, n_i ; i = 1, \dots, N \\ x_{ij}, y_i \in \{0,1\}, j = 1, \dots, n_i ; i = 1, \dots, N \end{cases}$$

- N is the number of families
- n_i is the number of jobs in family i
- c_{ij} is the profit of job j in family i
- a_{ij} is the time to process job j in family i
- f_i is the setup cost for family i $(f_i < 0)$
- d_i is the setup time for family i
- b is the time available for processing

Bilevel Knapsack Problem

$$(BKP)\quad \max_{x,y} \; f^1(x,y) = d^1 x + d^2 y \quad \longrightarrow \textit{Leader objective function}$$

$$\text{s.t.}\quad B^1 x + B^2 y \le b^1$$

$$x \in Z_+^{n_1}$$

$$\max_{y} \; f^2(y) = c\,y \quad \longrightarrow \textit{Follower objective function}$$

$$\text{s.t.}\quad A^1 x + A^2 y \le b^2$$

$$y \in \{0,1\}^{n_2}$$

Follower problem

Moore, Bard 1990 => $B^2 = 0$
Dempe, Richter 2000
Brotcorne, Hanafi, Mansi 2011

Relaxation

Definition : Let

(P) $\max\{ f(x) : x \in X\}$

(R) $\max\{ g(x) : x \in Y\}$

Problem R is a relaxation of P if

1) $X \subseteq Y$

2) $\forall\, x \in X : f(x) \leq g(x)$.

Properties

- $v(\mathrm{P}) \leq v(\mathrm{R})$

- Let x^* an optimal solution of R, if x^* is feasible for P and $f(x^*)=g(x^*)$ then x^* is an optimal solution of P

LP-Relaxation

Dantzig (1957)

$$(LP) \quad \begin{array}{ll} \max & cx \\ \text{s.t.} & Ax \leq b \\ & x \in [0,1]^n \end{array}$$

Properties :

- v(MKP) ≤ v(LP)

- Let x^* an optimal solution of LP,

if $x^* \in \{0,1\}^n$ then x^* is an optimal solution of MKP

MIP Relaxation

Hanafi & Wilbaut (06), Glover (06)

- **Let $x^0 \in \{0, 1\}^n$ and $J \subseteq N$**

$$MIP(P,J) \begin{cases} max & cx \\ s.c. & Ax < b \\ & x_j \in \{0,1\} \quad j \in J \\ & x_j \in [0,1] \quad j \in N - J \end{cases}$$

- **Remarks:**
  - ***MIP*(x^0,∅) = LP(P), *MIP*(x^0,N) = *P***
  - ***v*(MI*P*(x^0,*J*)) ≥ *v*(MI*P*(x^0,*J'*) if *J* ⊆ *J'***
- **Stronger bound : *v*(*P*) ≤ *v*(*MIP*(*P*,*J*)) ≤ *v*(*LP*(*P*))**

Semi-Continuous Relaxation

Hanafi & Mansi & Wilbaut (2009)

- Let $J \subseteq N$

$$\textbf{SCR(P, }\alpha\textbf{)} \begin{cases} \max cx \\ \text{s.c. } Ax \leq b \\ x_j \in [0 , \alpha_j] \cup [1-\alpha_j , 1] \;\forall\, j \in N \\ x_j \geq 0 \qquad\qquad j = n+1 \,..\, n' \end{cases}$$

- **Remarks**
  - ✓ $\alpha = e/2 \Rightarrow$ **SCR(P, α) ≡ LP (P)**
  - ✓ $\alpha = (0_J, e_{N-J}/2) \Rightarrow$ **SCR(P, α) ≡ MIP (P,J)**
  - ✓ $\alpha = 0 \Rightarrow$ **SCR(P, α) ≡ P**

Lagrangean Relaxation

Held & Krap (71), Geoffrion (74)

- **Lagrangean Relaxation : $\lambda \geq 0$**
  LR(λ) $\max\{cx + \lambda(b - Ax) : x \in X\}$
- **Lagrangean Dual :**
  (L) $\min\{v(LR(\lambda)) : \lambda \geq 0\}$

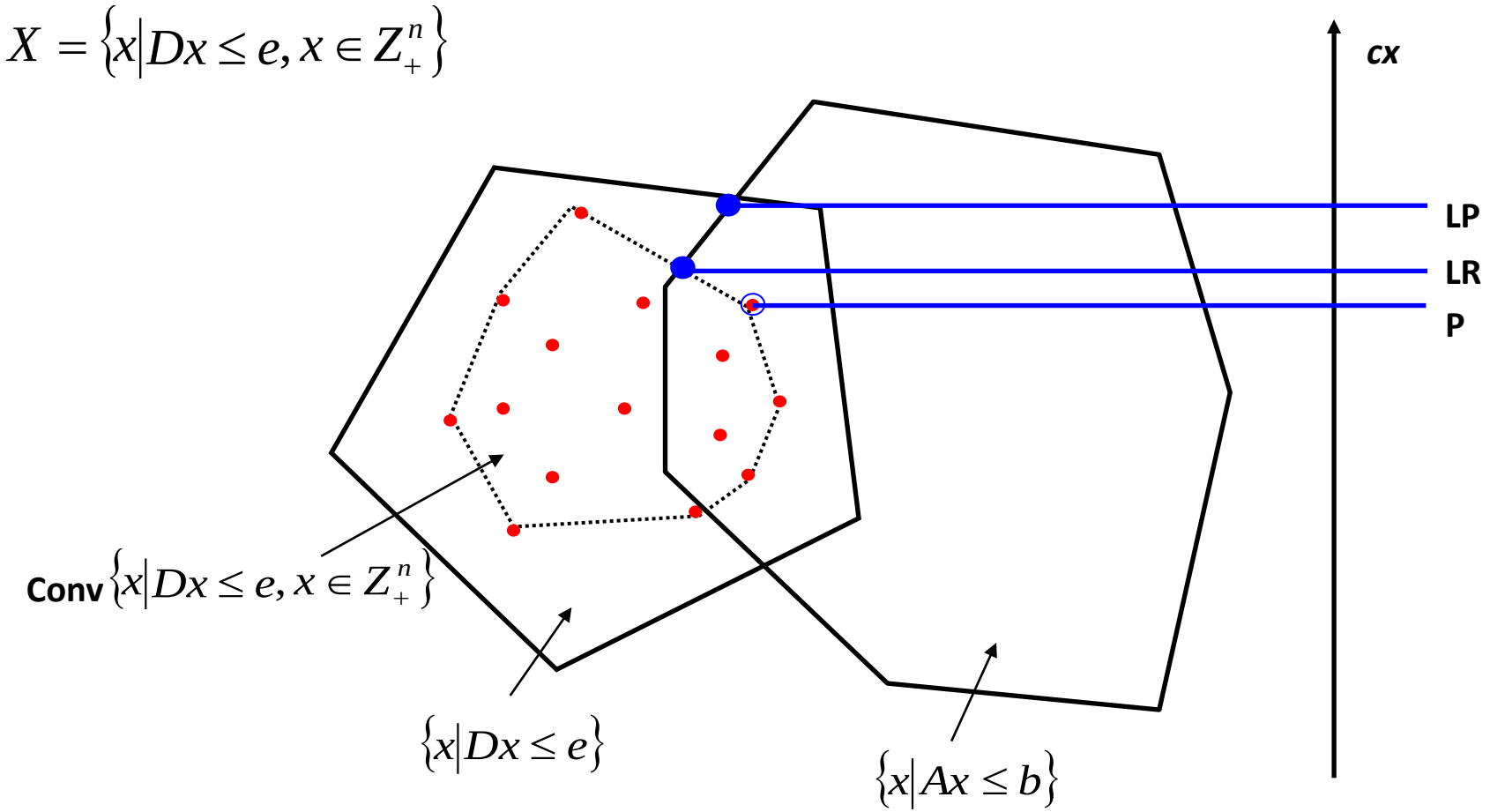
Properties :

- v(MKP) $\leq v$(L) $\leq v$(LR(λ))

- Let x^* be an optimal solution of LR(λ^*), if $\lambda^*(b - Ax^*) = 0$ and x^* is feasible for P then x^* is an optimal solution of P

Geometric Interpretation

$v(\mathrm{L}) = \max\{cx : Ax \leq b, x \in \mathrm{co}(X)\}$



Lagrangean Decomposition

Guignard & Kim (87)

$$Z = \max \quad cx$$
$$Ax \le b$$
$$\boxed{Dx \le e}$$
$$x \in Z_+^n$$

$$Z' = \max \quad cx$$
$$Ax \le b$$
$$\boxed{Dy \le e}$$
$$\boxed{x = y}$$
$$x \in Z_+^n$$
$$y \in Z_+^n$$

$$Z_{LD}(v) = \max \quad cx + v(y - x)$$
$$Ax \le b$$
$$Dy \le e$$
$$x \in Z_+^n$$
$$y \in Z_+^n$$

Lagrangean Decomposition

$$Z_{LD}(v) = \max \quad cx + v(y - x)$$
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Sub-problem 1

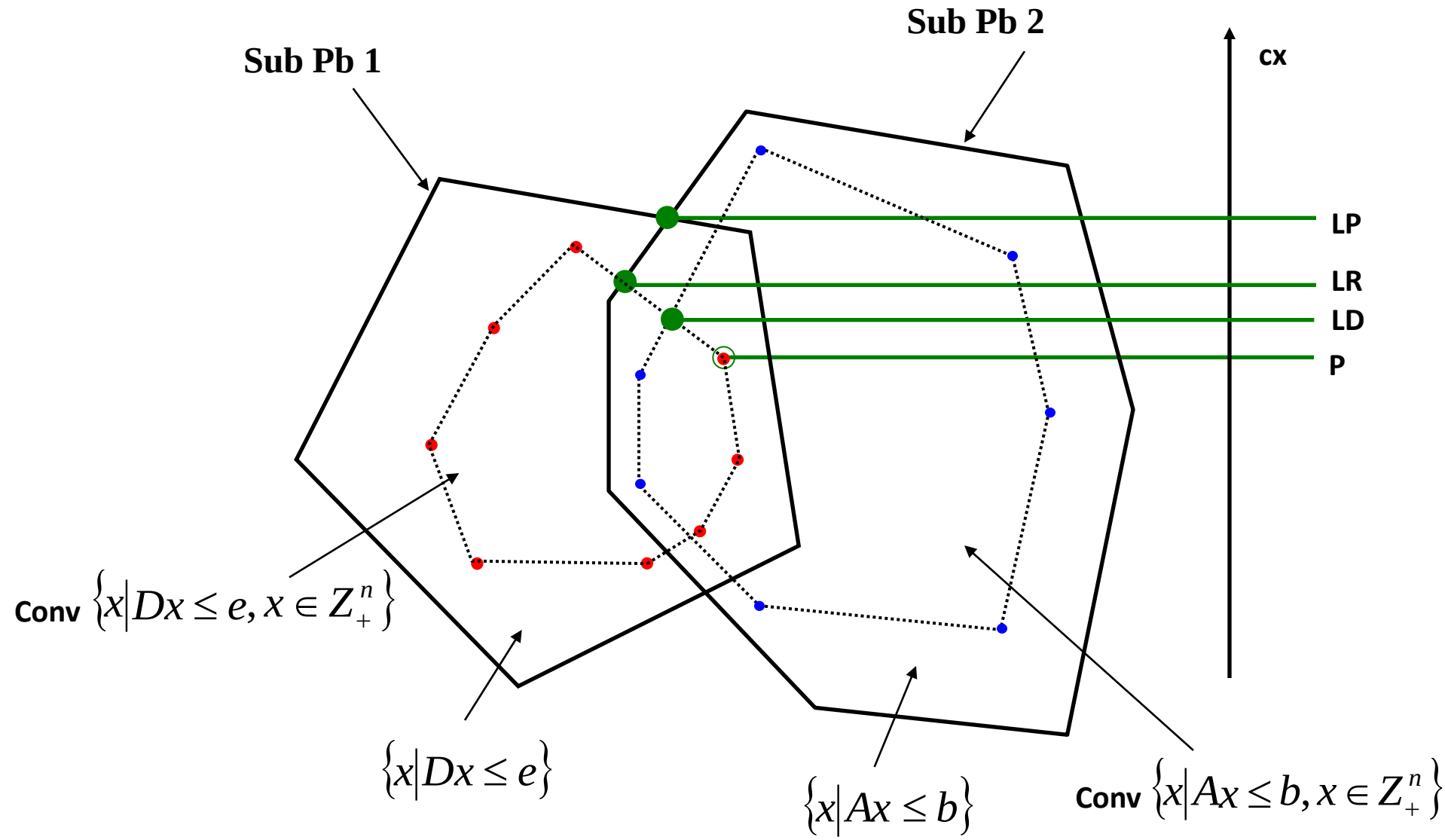
Sub-problem 2

$$Z_{LD1}(v) = \max \quad (c - v)x$$
$$Ax \le b$$
$$x \in Z_+^n$$

$$Z_{LD2}(v) = \max \quad vy$$
$$Dy \le e$$
$$y \in Z_+^n$$

$$Z_{LD} = \min_{v \ge 0} \quad (Z_{LD1}(v) + Z_{LD2}(v))$$

Geometric Interpretation



Theorem

$$v(P) \leq v(LD) \leq v(LR) \leq v(LP)$$

Surrogate Relaxation

- **Relaxation surrogate : $\mu \geq 0$**

  SR(μ) max{cx : μAx $\leq \mu$b, x $\in$ X}

Dual surrogate corresponding :

  (S) min{*v*(SR(μ)) : $\mu \geq 0$}.

Glover (65)

Proprieties :

- *v*(MKP) $\leq$ *v*(S) $\leq$ *v*(SR(μ))

- Let *x be an optimal solution of SR(μ),**

if *x is feasible for MKP then *x** is an optimal solution of MKP**

- (S) min{*v*(SR(μ)) : $\mu \in$ U = {$\mu \geq 0$: $||\mu|| = 1$}}

Composite Relaxation

Greenberg & Pierskalla (70)

- **Composite Relaxation : $\lambda, \mu \geq 0$**

$CR(\lambda, \mu)$ $\quad \max\{cx + \lambda(b - Ax) : \mu Ax \leq \mu b \text{ and } x \in X\}$

Composite Dual :

(C) $\quad \min\{v(CR(\lambda, \mu)) : \lambda, \mu \geq 0\}$

Remarks :

- $v(LR(\lambda)) = v(CR(\lambda, 0))$
- $v(SR(\mu)) = v(CR(0, \mu))$

Comparisons

$$v(P) \leq v(C) \leq v(S) \leq v(L) \leq v(LP)$$

Properties of Relaxation Functions

- **Lagrangean function $v(LR(\lambda))$ is convex and piecewise-linear if X is finite**
- **Surrogate function surrogate $v(SR(\mu))$ is quasi-convex and piecewise-linear if X is finite**
- **Composite function $v(CR(\lambda, \mu))$ is non convex, non quasi-convex**

Relaxation-based Heuristics

- $u = u°$, UB = $g(u)$, LB = $f(x^0)$
- Repeat
  - let $x^*(u)$ an optimal of the current relaxation $R(u)$
  - let $x(u)$ the projection of $x^*(u)$ an the feasible set of P
  - UB = min(UB, $v(R(u))$)
  - LB = max(LB, $f(x(u))$)
  - Update the current multiplier u

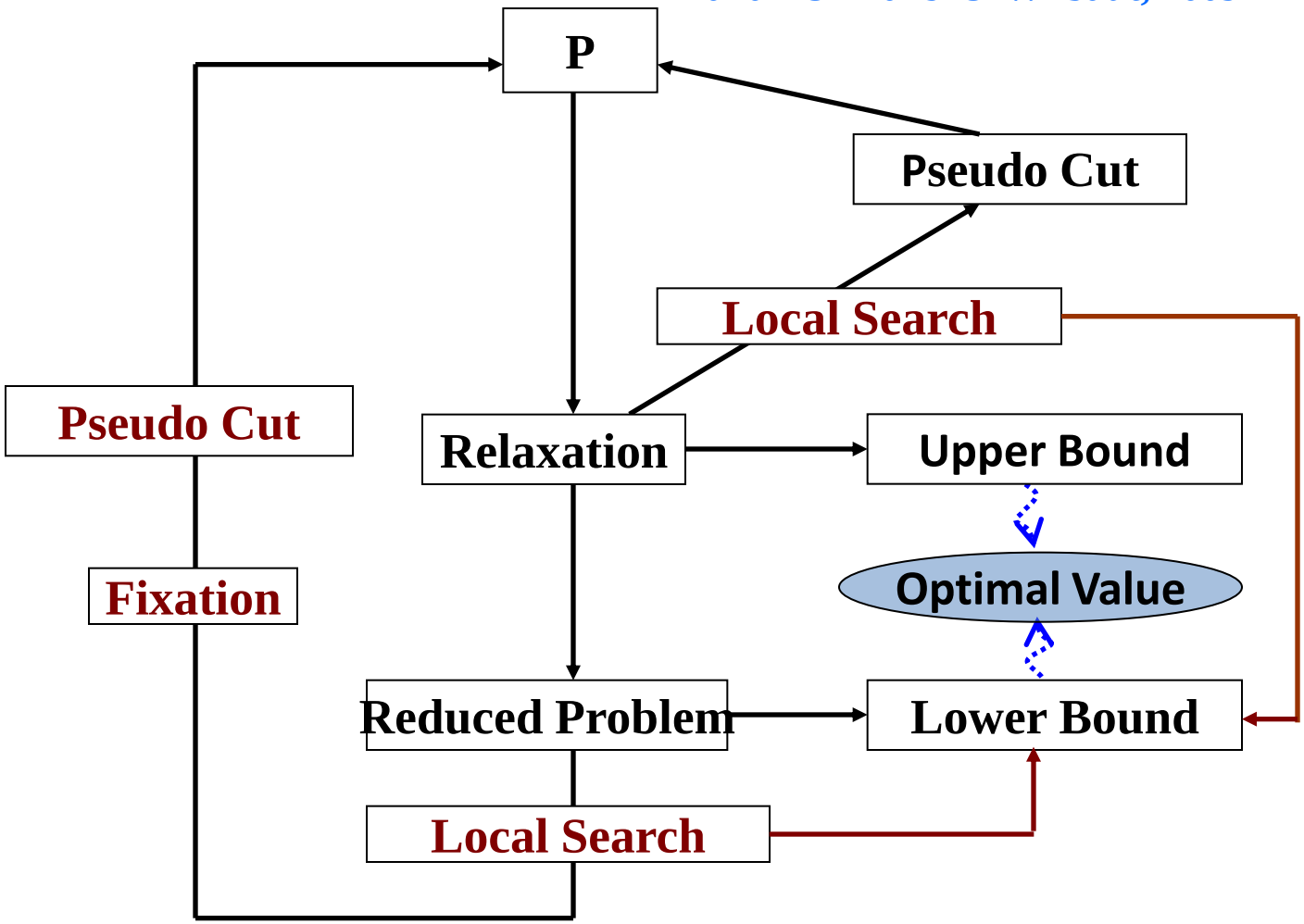
Relaxation-based Heuristics

Hanafi & Wilbaut, 2006

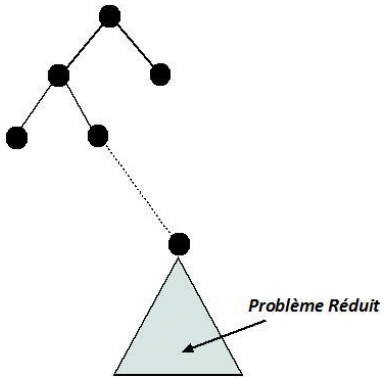
- **Step 1:** Solve one or more **relaxations** of the current problem *P* to generate one or more constraints.
- **Step 2:** Solve one or more **reduced problems** induced by the optimal solution(s) of the previous relaxation(s) to obtain one or more feasible solution(s) of the initial problem.
- **Step 3:** If a stopping criteria is satisfied then return the best lower bound. Else add a generated **constraint**(s) to the problem *P* and return to step 1.

Relaxation-based Heuristics

Hanafi & Mansi & Wilbaut, 2009



Reduced Problem



- Let $x^0 \in \{0, 1\}^n$ and $J \subseteq N$, $x_J = (x_j)_{j \in J}$

$$P(x^0, J) = (P| \; x_J = x^0{}_J)$$

- **Remarks :**
  - $P(x^0, \varnothing) = P$, $v(P(x^0, J)) \geq v(P(x^0, J'))$ if $J \subseteq J'$
  - $x_J = x^0{}_J \Leftrightarrow (e_J - 2\, x^0{}_J) x_J + e_J\, x^0{}_J = 0$
  - $|J| = 1$ If $v(P(e - x^0, \{j\})) \leq cx^0$ then $x^*_j = x^0_j$, x^* an optimal solution

Pseudo-Cut

Let y be a vector in $\{0, 1\}^n$, the following inequality (*) cuts off the solution y without cutting off any other solution in $\{0, 1\}^n$

$$\sum_{j \in J^1(y)} x_j - \sum_{j \in J^0(y)} x_j \leq |J^1(y)| - 1 \qquad (*)$$

where $J^0(x) = \{j \in N : x_j = 0\}$ $J^1(x) = \{j \in N : x_j = 1\}$.

Example:

$$y = (1\ 0\ 1)$$
$$y_1 - y_2 + y_3 \leq 1$$

y_1	0	0	0	0	1	1	1	1
y_2	0	1	0	1	0	1	0	1
y_3	0	0	1	1	0	0	1	1
$y_1 - y_2 + y_3$	0	-1	1	0	1	0	2	1

(*) is called canonical cut in Balas & Jeroslow, 1972

Partial Pseudo Cuts

Let x' be a *partial solution* in $\{0, 1, *\}^n$, the inequality

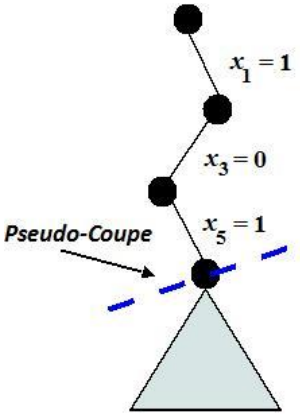
$$d_J(x, x') = (e_J - 2\,x'_J)x_J + e_J\,x'_J \geq 1$$

cuts off x' and all partial solution dominated by x'.

$$J = \{j \in N: x'_j \in \{0,1\}\},\ x_J = (x_j : j \in J)$$

$x' = (1\ *\ 0\ 1\ *)$, $J = \{1, 3, 4\}$

$-x_1 + x_3 - x_4 \geq -1$



Remarks :

- **$d_N(x, x')$ = Hamming distance over $\{0, 1\}^n$**

Iterative LP-based heuristic (ILPH)

Let x^* a feasible solution of P

$v^* = cx^*$; $Q = P$; *stop = False*;

while (*stop = False*) do

Let x^R be an optimal solution of $R(Q)$; J = $J(x^R)$;

Let x^0 be an optimal solution of $(Q / (e_J - 2\, x^R{}_J) x_J \leqq k - e_J\, x^R{}_J)$

if $cx^0 > v^*$ then $x^* = x^0$; $v^* = cx^0$

$Q = (Q \mid (e_J - 2\, x^R{}_J) x_J \geq k + 1 - e_J\, x^R{}_J)$

if $\lfloor cx^R - v^* \rfloor < 1$ then *stop = True*

end while

Theorem : Finite Convergence

The algorithm *ILPH* converges to an optimal solution of the problem in a finite number of iterations ($\leq 3^n$).

Proof :

- $v(P) = max(v(P/dx \leq d_0), v(P/dx \geq d_0 + 1))$
- $d = (e_J - 2\, x^R_J, 0)$ and $d_0 = -\, e_J\, x^R_J$
- **Number of partial solutions = $|\{0, 1, *\}^n| = 3^n$**

Convergence of IPLH

$GK9$ (n=30 m=10)				OR-5.100-4 (n=100 m=5)			
$iter$	$v(LP)$	cx	$\lvert F(x^{LP})\rvert$	$iter$	$v(LP)$	cx	$\lvert F(x^{LP})\rvert$
1	380.3	336	6	1	23724.1	22554	5
2	379.7	368	7	2	23722.7	22983	5
3	379.61	360	8	40	23687.4	23447	18
11	**376.99**	372	13	41	23687.3	**23534**	20
12	376.53	364	7	100	23651.1	23497	24
13	376.42	**376**	9	292	**23534.6**	23497	46

$F(x)$: set of fractional variables of solution x

CPU ≈ 400sec
(Cplex ≈ 10sec)

⇒ Replace the optimal condition by a total number of iterations

Results for MKP : $n = 500$

m	α	Vasquez & Vimont	IRH	$IIRH$
5	0.25	120629.2	120629.2	120630.3
	0.5	219512.7	219512.7	219512.7
	0.75	302363.4	302363.4	302363.4
10	0.25	118628.6	118625.6	11830.8
	0.5	217327.1	217328.3	217330
	0.75	302602.7	302598.9	302604.6
30	0.25	115623.7	115599.5	115605.7
	0.5	216274.7	216255.8	216262.5
	0.75	302446.5	302424.4	302432.5

Average computational time **70 hours** **2 hours**

1 line = average / 10 instances

MKP : Summary (n = 500)

m	#Improvement	#Equality	#Less	Average gap(%)
5	**1**	29	0	0
10	**8**	16	6	0
30	**3**	12	15	0.01

1 **line = results** for 30 instances

Results for MMKP

	C&H			CPLEX		
	#Best	#New Best	Total	#Best	#New Best	Total
HIPL	9	7	16	10	21	31
HIMIP	8	8	16	11	17	28
HIRSC	7	3	10	9	11	20
HIRI	7	5	12	10	19	29

Bi-Objective Max Min Knapsack Problem

n	*VPT*			*ILPH*			
	cpu	*#opt*	*%F*	*cpu*	*#opt*	*%F°*	*%F*
100	32.72	10	07	< 1	10	07	47
200	169.45	9	06	< 1	10	07	50
300	450.18	5	01	10	10	01	37
400	183.18	8	09	8	08	09	63
500	260.45	7	07	15	10	08	61
600	164.00	8	29	11	09	29	56
700	518.09	4	03	19	10	03	56
800	331.27	6	15	30	10	16	66
900	249.09	7	15	18.18	09	16	66
1000	424.54	6	05	166.36	08	06	49

CPU < 900 s

New Hybrid VNDS with IRH

VNDS-IRH: problem P ; initial solution x^*; d ; k_{vnd}

Add objective cut: P = (P | $cx > cx^*$)

While (not Stop1) do

$\bar{x}$ = optimal solution for LP(P)

if ($\bar{x}$ is integer) then $x^* = \bar{x}$, break

Reorder $\delta_j = |x^*_j - \bar{x}_j|$, $j \in B$, so that: $\delta_1 \leq \delta_2 \leq \ldots \leq \delta_{|B|}$

$n_d = |\{j \in B : \delta_j \neq 0\}|$; $k_{step} = [n_d/d]$; $k = |B| - k_{step}$;

While ((not Stop2) and $k \geq 0$) do

J = {1,...,k}; x' = MIPSOLVE(P(x^*,J),x^*(N-J))

if ($cx' > cx^*$) then x^* = VND(P,k_{vnd},x'), break

if ($k - k_{step} > |B| - n_d$) then $k_{step} = \max\{[k/2], 1\}$

$k = k - k_{step}$

Update Stop2

x' = MIPSOLVE(P($\bar{x}$)); $x^* = \text{argmax}\{cx', cx^*\}$

Add pseudo-cut : P = (P | $\delta(J(\bar{x}), x, \bar{x}) \geq 1$)

Update Stop1

Return x^*

Average Results for the OR-Library

Best known values:

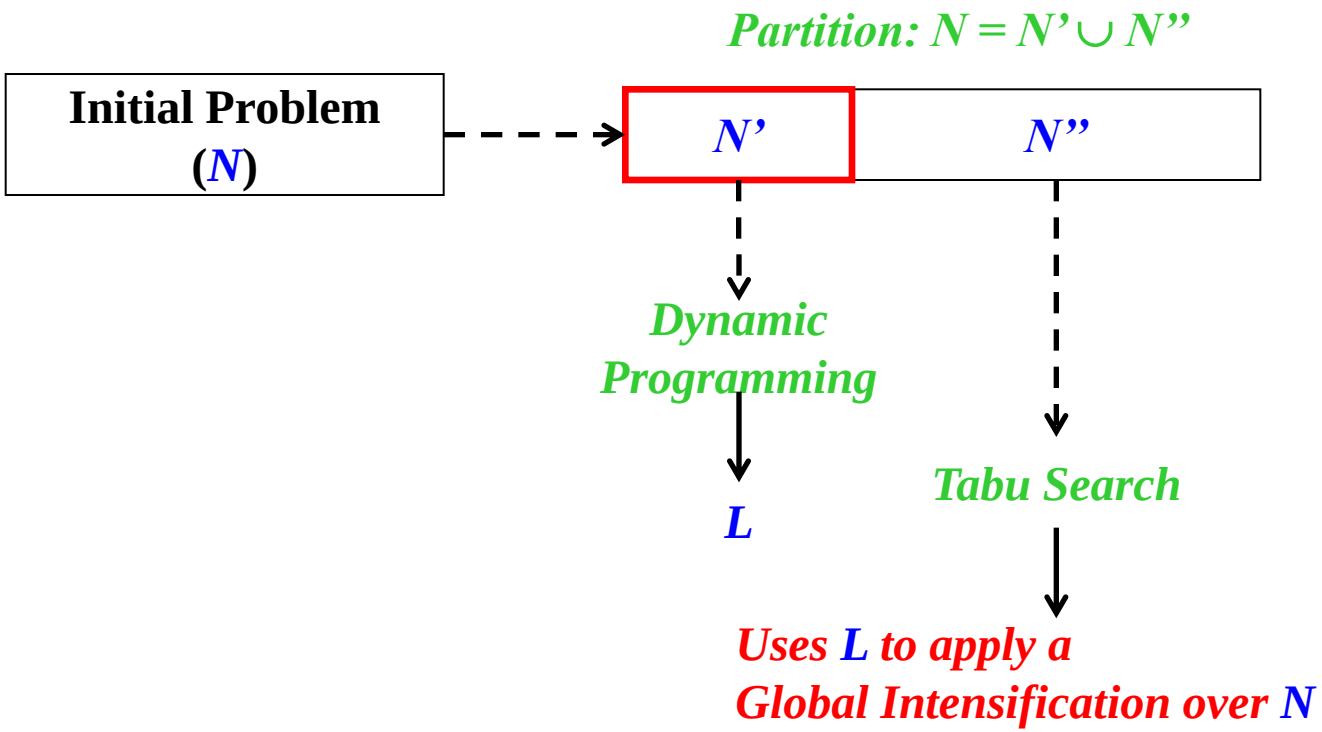
Boussier & Vasquez & Vimont & Hanafi & Michelon (2010)

Hanafi & Wilbaut (2009) Vasquez & Vimont (2005)

Vimont & Boussier & Vasquez (2008) Wilbaut & Hanafi (2009)

		VNDS-IRH	VNDS-IRH2	VNDS-IRH3
10.500	#best	6	5	4
	Avg. Gap	0.009	0.009	0.012
	Avg. CPU*	1703	2057	1508
30.250	#best	21	17	14
	Avg. Gap	0.009	0.013	0.020
	Avg. CPU*	1449	1298	1040
30.500	#best	2	1	3
	Avg. Gap	0.032	0.030	0.031
	Avg. CPU*	2063	1925	2270

Basic Idea of the "Global Intensification" Scheme



➡ Add a second level of intensification in TS

Application to the 0-1 MKP

- **Formulation**

$$(\text{MKP}) \left[\begin{array}{lll} \max & \sum_{j=1}^{n} c_j x_j & \\ \text{s.t.:} & \sum_{j=1}^{n} a_{ij} x_j \leq b_i & \forall i \in M = \{1, \ldots, m\} \\ & x_j \in \{0,1\} & j \in N = \{1, \ldots, n\} \end{array} \right.$$

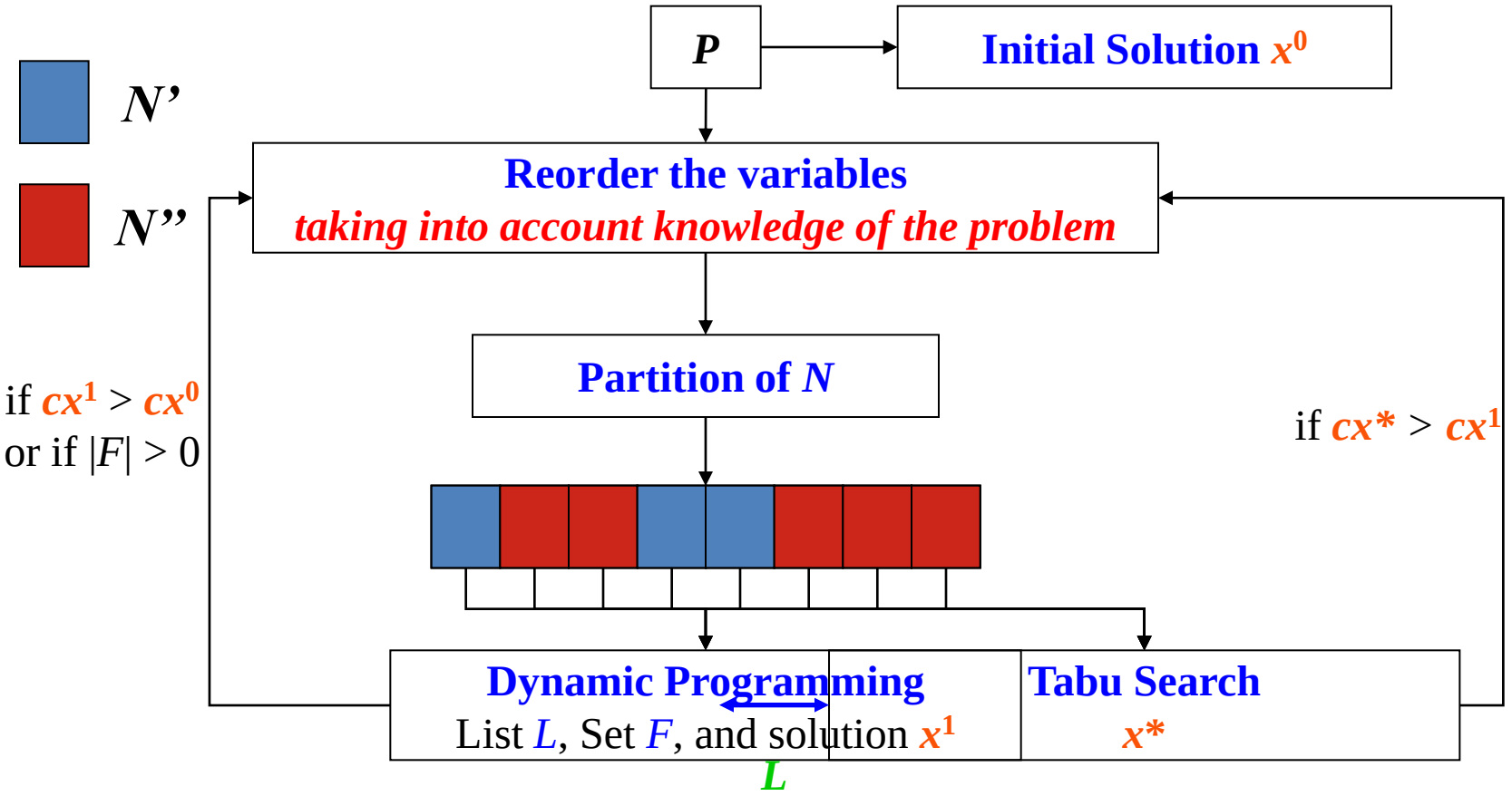
- **Dynamic Programming ($0 \leq k \leq n$ and $0 \leq \beta \leq b$)**

$$(\text{MKP}(k,\beta)) \left[\begin{array}{lll} \max & \sum_{j=1}^{k} c_j x_j & \\ \text{s.t.:} & \sum_{j=1}^{k} a_{ij} x_j \leq \beta_i & i = 1, \ldots, m \\ & x_j \in \{0,1\} & j = 1, \ldots, k \end{array} \right.$$

$$v(\text{MKP}(k,\beta)) = f(k,\beta) = \left\{ \begin{array}{ll} f(k-1,\beta) & \text{if } a_k \not\leq \beta \\ \max\{f(k-1,\beta), c_k + f(k-1,\beta-a_k)\} & \text{if } a_k \leq \beta \end{array} \right.$$

Global Intensification

Let P be an instance of MKP



Global Intensification

- Why (and how) Reordering the variables?

➡ To set some variables at their optimal values

- Let x^0 be an initial solution of P
- Let $u_j \geq v(MKP \,|x_j = 1 - x_j^0)$ for all j in N
- **Lemma**

$$\text{if } v(MKP \,|x_j = 1 - x_j^0) \leq c^T x^0 \text{ then } x_j^* = x_j^0$$

- Reorder the variables such that:

$$u_1 \geq u_2 \geq \ldots \geq u_n$$

Global Intensification

- Property 1 (**Optimality**):
  Let $l_j = v(MKP \,|\, x_k = x_k^0, \forall k = j+1, \ldots, n)$
  if $\exists j \in \{1, 2, \ldots, n\}$ such that $l_j \geq u_{j+1}$
  then $v(MKP) = l_j$

- Proof: let x^* be an optimal solution and $j \in N$ such that $l_j \geq u_{j+1}$

  First case: if $x_k^* = x_k^0, \forall k = j+1, \ldots, n$ => OK

  Second case: if $\exists k \in \{j+1, \ldots, n\}, x_k^* \neq x_k^0$

$$u_{j+1} \geq u_k \text{ since } u_1 \geq \ldots \geq u_n$$

$$u_k \geq v(MKP \,|\, x_k = 1 - x_k^0) = v(MKP) \text{ since } x_k^* \neq x_k^0$$

$$l_j \geq u_{j+1} \geq v(MKP)$$

Global Intensification

- Remark: $l_1 \leq l_2 \leq \ldots \leq l_n$ (1)
- Property2 (**Reduction**):

  If $\exists h, k$ with $h < k < n$ and $l_h \geq u_k$
  then $x_j^* = x_j^0, \forall j \in \{k, \ldots, n\}$

- Proof: suppose $\exists\ h < k < n$ with $l_h > u_k$ (H)

  then: $h < k$ $\Rightarrow h \leq k - 1$

  $\Rightarrow l_h \leq l_{k\text{-}1}$ from (1)

  $\Rightarrow u_k \leq l_h \leq l_{k\text{-}1}$ from (H)

  $\Rightarrow v(\text{MKP}) = l_{k\text{-}1}$ (Prop.1)

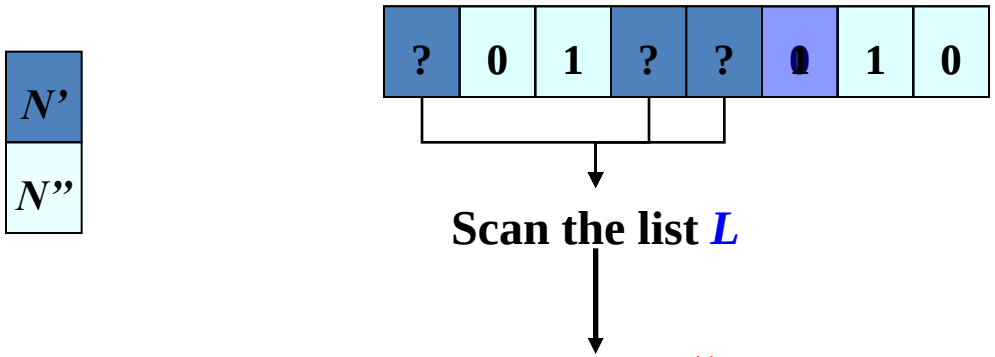
Global Intensification

- Using the List L in Tabu Search
  - **Neighborhood** of the current solution *x''* on *N''*

➡ Drop / Add one item

$$V(x'') = \{y \in \{0,1\}^{|N''|} \,\Big|\, Ay \leq b \text{ and } \sum_{j \in N''} \left|x''_j - y_j\right| \leq 1\}$$

  - **Evaluation of a move**



$$max\{v : \beta \geq \sum_{j \in N''} a_j x''_j, (v,\beta) \in L\} + \sum_{j \in N''} c_j x''_j$$

Global Intensification

- Experiments over OR-Library instances
  - The solution quality **increases during the search** (initial solution, dynamic programming, tabu search)
  - The Global Intensification process obtains **better** results than a "*simple*" **Tabu Search**
  - The results were **better** than those obtained by **CPLEX**
  - Reduction rules were **efficient when $m << n$**

Global Intensification: Conclusions

- Introduction of a **second level of intensification** in Tabu Search
- The Global Intensification **can be applied** for a class of combinatorial optimization problems, whose the structure is adapted to Dynamic Programming treatment
- **Efficient cooperation** between Dynamic Programming and Tabu Search (for the MKP)

Decomposition

Glover 1965, Vasquez, 2000, Hanafi et al. 2011

Let x^* be best known solution

$$X = \{x : Ax \leq b, cx \geq cx^*+1\}$$

$$k_{min} = \min\{ex : x \in X \cap [0,1]^n\}$$

$$k_{max} = \max\{ex : x \in X \cap [0,1]^n\}$$

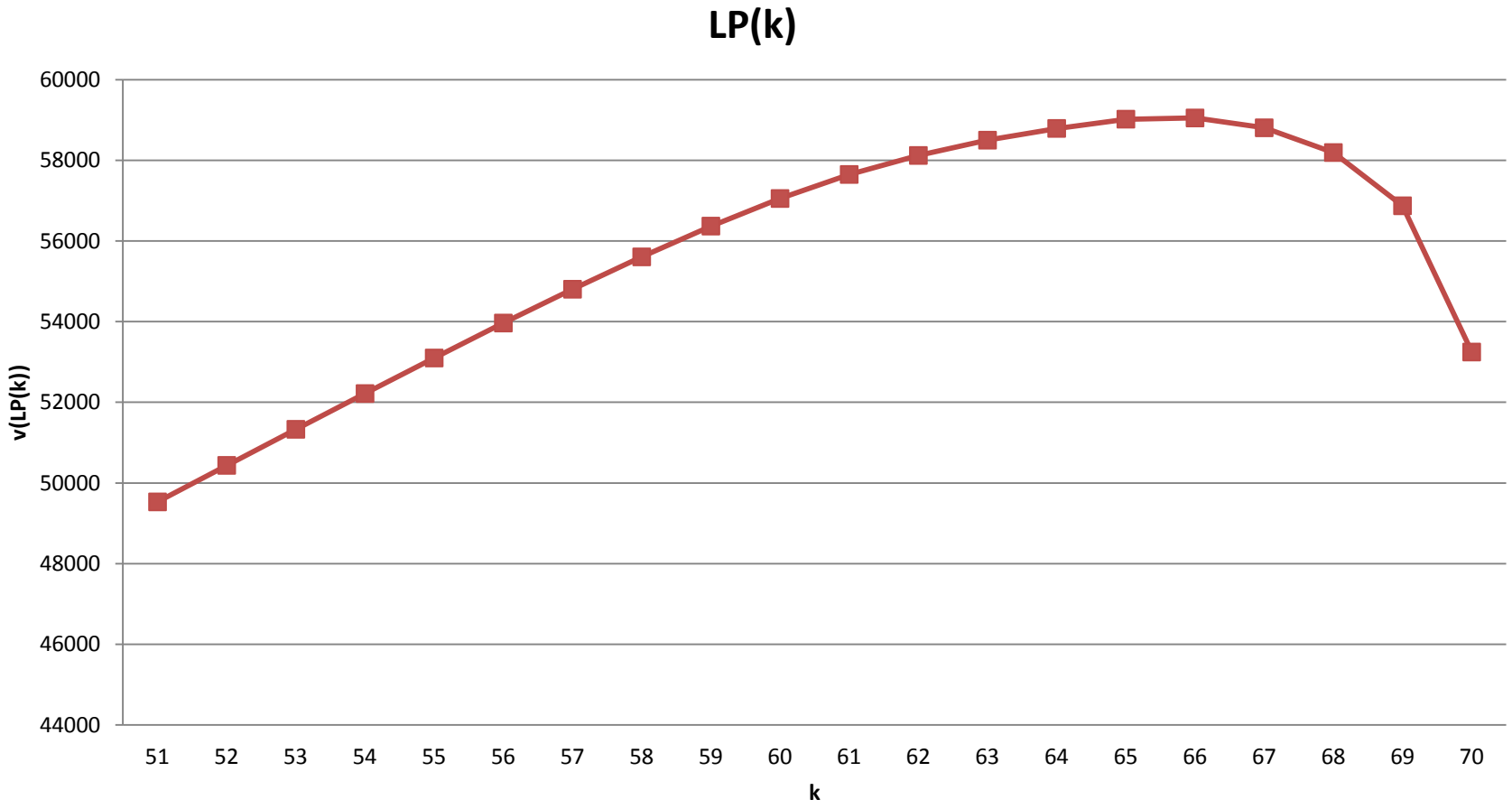
$$\text{MKP}(k)\ \max\{cx : ex = k\ ,\ x \in X \cap \{0,1\}^n\}$$

We have:

$$v(\text{MKP}) = \max\{v(\text{MKP}(k)) : k = \lceil k_{min} \rceil, \ldots, \lfloor k_{max} \rfloor\}$$

$$\text{LP}(k) = \text{LP}(\text{MKP}(k))$$

$v(\mathrm{LP}(k))$ is concave



Tabu Search with Relaxation for MKP

Let $k = \lfloor ey \rfloor$ and $h = \lceil ey \rceil$ where y = R(MKP); $x^* = 0$;

do{

Stop = False; y = Relaxation(MKP(k));

if $cy > cx^*$ then

x = Tabu-Search(MKP(k), y); $k = k - 1$;

$x^* = argmax\{cx, cx^*\}$; Stop = True;

y = Relaxation(MKP(h));

if $cy > cx^*$ then

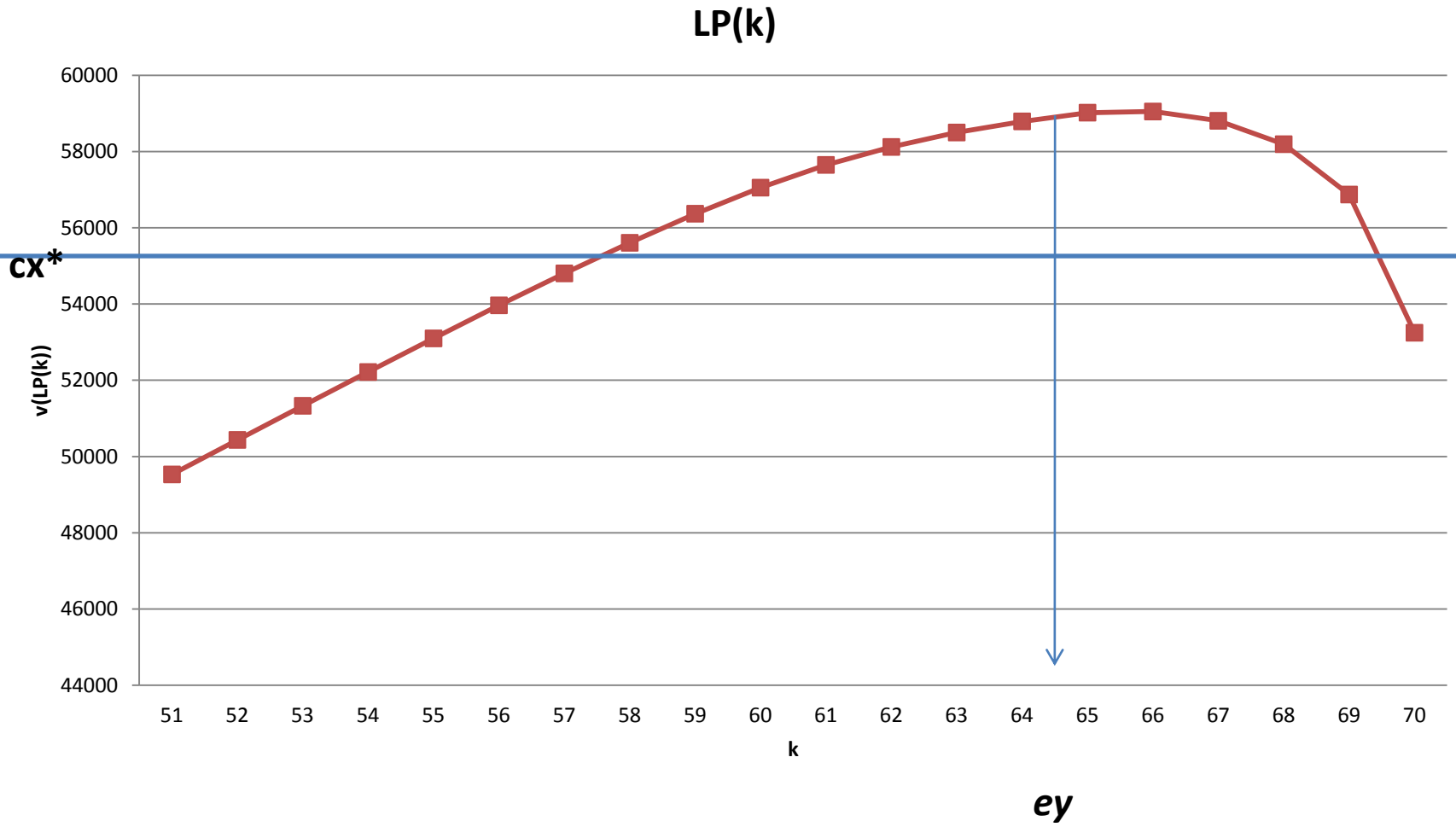
x = Tabu-Search(MKP(h), y); $h = h + 1$;

$x^* = argmax\{cx, cx^*\}$; Stop = True;

}while Stop;

Relaxation = {LP or MIP}

Hyperplanes exploration order



Tabu Search for MKP(k)

Neighborhood Vasquez & Hao, 2001; Vasquez & Vimont, 2005;

- $y = \mathrm{R}(k)$; $\sigma = 2(ey - k)$
- $N(x) = \{x' \in \{0,1\} : \|x - x'\|_1 = 2$
  $\|x - y\|_1 \leq \sigma\ ,\ ex' = k\}$

Dynamic Tabu List Glover, 1990

Reverse Elimitation Method Dammeyer & Voß, 1993; Glover & Laguna, 1997; Hanafi & Fréville, 1999;

Aspiration

– New best solution even $\|x' - y\|_1 > \sigma$

Results CB30.500-0

```
CB30.500_0 k=130 hwf=0 hwb=0 ub=116601.4

.     2 115332      0 1224 1653     0 sec.

.     2 115604 115332 1127 1249     0 sec.

*    40 115637 115637 1326 1351     0 sec.

*     2 115653 115653 1492 1517     0 sec.

*     4 115688 115688 1656 1681     0 sec.

*     6 115695 115695 1740 1765     0 sec.

*    26 115708 115708 1781 1806     1 sec.

.   468 115713 115708 1722 1806     3 sec.

.  1014 115809 115713 1641 1747     4 sec.

.     2 115832 115809 1594 1666     5 sec.

*  9906 115862 115862 1752 1777    28 sec.

.   296 115869 115862 1750 1777    38 sec.

*   108 116056 116056 1821 1846    54 sec.
```

Results CB30.500

	lb	k
cb30.500_0	116056	130
cb30.500_1	114810	128
cb30.500_2	116721	128
cb30.500_3	115329	127
cb30.500_4	**116565**	**128**
cb30.500_5	115741	131
cb30.500_6	114181	128
cb30.500_7	114348	128
cb30.500_8	115419	128
cb30.500_9	117116	128
cb30.500_10	218104	251
cb30.500_11	214648	251
cb30.500_12	215978	250
cb30.500_13	217910	251
cb30.500_14	215689	251

	lb	k
cb30.500_15	215890	253
cb30.500_16	215907	253
cb30.500_17	216542	254
cb30.500_18	217340	253
cb30.500_19	214739	252
cb30.500_20	301675	375
cb30.500_21	300055	374
cb30.500_22	305087	375
cb30.500_23	302032	375
cb30.500_24	304462	375
cb30.500_25	297012	374
cb30.500_26	303364	373
cb30.500_27	307007	377
cb30.500_28	303199	375
cb30.500_29	300572	376

Some Related Works

- Canonical Cuts on the Unit Hypercube, Balas & Jeroslow, 1972
- 0-1 IP with many variables and few constraints, Soyster, Lev, Slivka, 1978
- A hybrid approach to discrete mathematical programming, Marsten, Morin, 1978
- A new hybrid method combining exact solution and local search, Mautor, Michelon, 1997
- Parametric Branch and Bound, Glover, 1978
- Large neighborhood search, Shaw, 1998
- Local branching, Fischetti, Lodi, 2002

Some Related Works

- Relaxation induced neighbourhood search, Danna, Rothberg, Pape, 2003
- Adaptive memory projection method, Glover, 2005
- Feasibility Pump, Fischetti, Glover, Lodi, 2005
- Relaxation guided VNS, Puchinger, Raidl, 2005
- Global intensification using dynamic programming, Wilbaut, Hanafi, Fréville, Balev, 2006
- Convergent Heuristics for 0-1 MIP, Wilbaut, Hanafi, 2006
- Variable Neighbourhood Decomposition Search, Lazic, Hanafi, Mladenovic, Urosevic, 2009
- Inequalities and Target Objectives in Metaheuristics for MIP, Glover, Hanafi, 2009