Exploring Integral Quantization Across Diverse Phase-Space Geometries: A Quantum Information Perspective

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Context

A general quantum system can be described either by using a density matrix or a phase space distribution. The advantageous of a phase-space description is that the full information about the quantum state is represented into a single distribution. Depending on their degree of freedom and symmetries, the phase space distribution takes values into different manifold that reflects the intrinsic Lie algebra structure that governs the description of the dynamics of the quantum system. These manifolds capture the underlying symmetries and conserved quantities of the system, allowing for a deeper understanding of its behavior and evolution. This geometric approach not only simplifies the visualization of quantum states but also facilitates the analysis of quantum dynamics and the identification of important physical properties [1,2,3].

The phase space distribution of a quantum system is also an appealing tool for visualization. Observing the positivity of the Wigner distribution (in any phase space geometry) serves as an indicator of the efficient simulation (meaning in polynomial time by a classical computer) of the quantum state. Another phase space distribution that represents the quantum system is the overlap of coherent states with the state of interest called the Majorana representation: the zeros of this distribution define the corresponding "stars." There are now strong indications that characterizing the quantumness of a quantum state based on the distribution of zeros in the stellar distribution is relevant [2,4,5]. A hierarchy of the number of zeroes can be established, and it appears that states without any zeros are the most classical states, in the sense that their use does not provide any computational or metrological advantage. This insight not only aids in identifying the quantum nature of a state but also underscores the practical importance of understanding its phase space distribution, especially when considering applications in quantum computing and precision metrology.

Objective

Our objective will be to build the stellar representation corresponding to coherent states of different Lie groups (as SU(1,1), SO(3)) and Hopf group (ϕ-deformed SU(2)) based on the covariant integral quantization approach [2,6,7]. This representation is relevant for studying the dynamics of experimentally relevant quantum optics respectively two-mode quantum fields, rotation spectra of molecules, and quantum optical states generated by non-linear processes. We will then use the developed phase space distributions taking values in various geometries - that correspond to the adapted representations of different quantum systems - for quantifying their quantumness, metrological and computation performances [4,5]. Our mathematical study approach will be grounded in quantum optics experiments, allowing us to bridge the theoretical framework with practical laboratory setups and measurements. By directly linking our mathematical investigations to experimental observations (as in [1,2]), we aim to develop a comprehensive understanding of quantum optical phenomena, leading to practical applications and insights that can drive advancements in quantum technologies and fundamental physics.

Profile of the candidate: Ph.D. in quantum information, quantum optics or in mathematical physics (group theoretical methods in physics, quantum groups, integral quantization)

Keywords: Phase space distribution, quantum groups, integral quantization, group theoretical methods in physics, quantum information, quantum metrology, quantum computing

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