On Optimal Design of Charging Stations for Electric Vehicles

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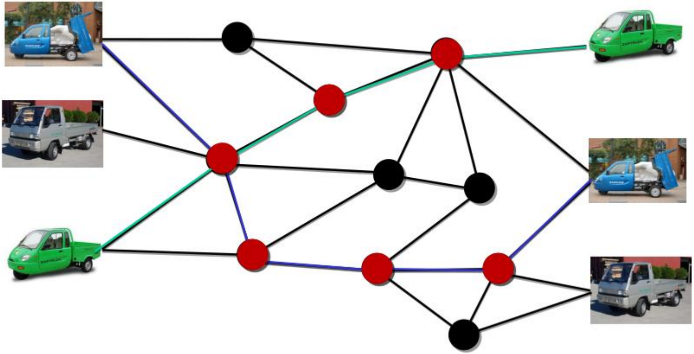
Motivation: E-vehicles transporting goods from A to B



1. Where to place charging stations, and how many?
2. Where to use shortcuts (and pay a road toll)?

Application Areas: E-Mobility and Design of Telecom Networks

- **E-Mobility:** E-cars need to recharge after traveling a certain distance
- **Congestion charging mechanisms**

- Goal: Fulfill the demands while minimizing the cost of road tolls and charging stations.

Application Areas: E-Mobility and Design of Telecom Networks

- **E-Mobility:** E-cars need to recharge after traveling a certain distance
- **Congestion charging mechanisms**

- Goal: Fulfill the demands while minimizing the cost of road tolls and charging stations.

Network Design: Signals can only be transmitted over limited distances (signal deterioration). Signal regenerators to be placed, or additional lines to be purchased.

e4share Project

e4-share

Models for Ecological, Economical, Efficient, Electric Car-Sharing

Consortium: universität wien, AIT Austrian Institute of Technology, ULB, Alma Mater Studiorum Università di Bologna, iC consulenten

Outline

Problem Definition

- Undirected graph $G = (V, E)$
- **Edge costs w** : $\mathbf{E} \to \mathbb{Q}_{\mathbf{0}}^{+}$ and **edge lengths d** : $\mathbf{E} \to \mathbb{Q}^{+}$
- Maximum distance $d_{\max} \in \mathbb{N}^{+}$
- **Relay costs c** : $\mathbf{V} \to \mathbb{Q}^{+}$
- Set of **node pairs** $\mathcal{K}$ that need to communicate

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- An s-t path p is feasible iff its length $d(p) \leq d_{\max}$. Otherwise, relays $\{r_1, \ldots, r_k\}$ to be placed so that

$$p = (\mathbf{s}, p_1, r_1, p_2, r_2, \ldots, r_k, p_{k+1}, \mathbf{t}),$$

and length of every subpath p_i is $d(p_i) \leq d_{\max}$.

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and length of every subpath p_i is $d(p_i) \leq d_{\max}$.

Goal:

Choose a subset of relays and edges with minimal total costs s.t. there exists a feasible path for every pair in $\mathcal{K}$.

Example

Graph $G = (V, E)$, $d_{\max} = 4$

$\mathcal{K} = \{(0,3),(0,4),(2,5),(3,4)\}$

Free (existing) edges E_0:

$E^0 = \{e | w(e) = 0\}$

Augmenting edges E^*:

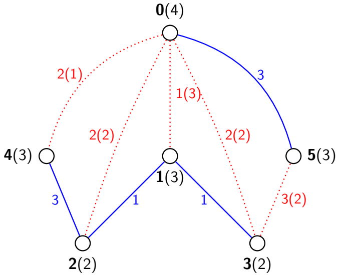
$E^* = \{e | w(e) > 0\}$

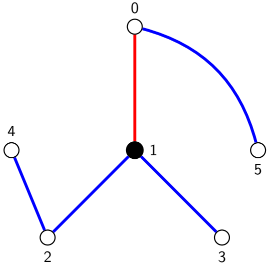
nodes **index (relay cost)**

edges **cost (distance)**

Optimal solution:

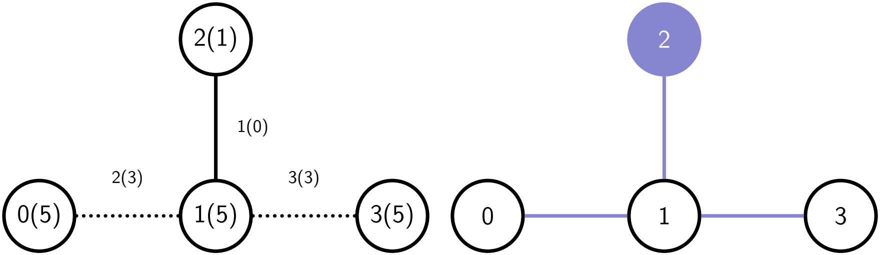
$1 + 3$





Cycle Property

- $d_{max} = 4$, $\mathcal{K} = \{(0,3)\}$



Property 1

In an optimal solution there exists for every pair $(u, v) \in \mathcal{K}$ a path from u to v visiting each relay **at most once**.

Property 2

In an optimal solution there exists for every pair $(u, v) \in \mathcal{K}$ a path from u to v visiting each non-relay node **at most twice**.

Cycle Property

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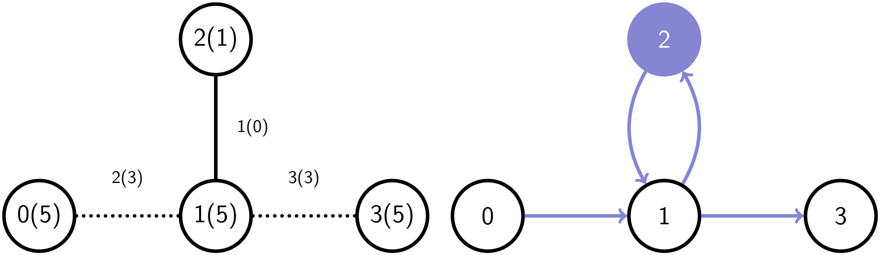

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Outline

Previous Work

E. A. Cabral, E. Erkut, G. Laporte, and R. A. Patterson. The network design problem with relays. *European Journal of Operational Research*, 180(2):834–844, 2007.

A. Konak. Network design problem with relays: A genetic algorithm with a path-based crossover and a set covering formulation. *European Journal of Operational Research*, 218(3):829–837, 2012.

S. Lin, X. Li, K. Wei, and C. Yue. A tabu search based metaheuristic for the network design problem with relays. In *Service Systems and Service Management (ICSSSM), 2014 11^{th} International Conference on*, pages 1–6, 2014.

Our Contribution

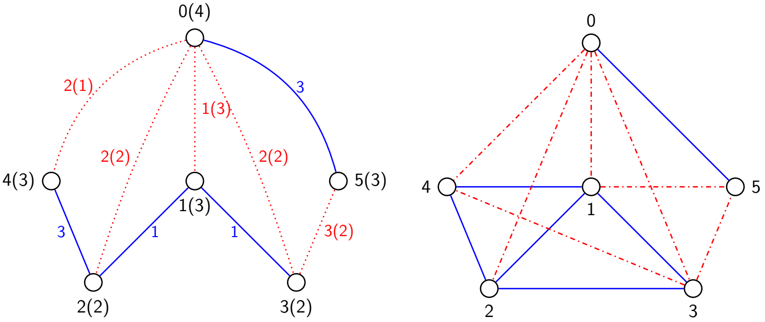
- **Our contribution:** We introduce 3 MIP models derived on the communication graph.
- Derive two Branch-and-Price and a Branch-Price-and-Cut algorithm.
- Outperform heuristics on smaller instances from the literature and solve them to optimality.
- Provide best known UBs for unsolved instances.

Outline

Communication Graph: Idea

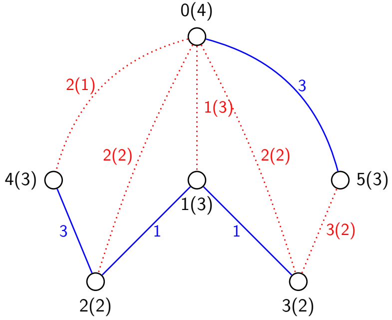
Used by Chen et al. for solving the Regenerator Location Problem. Main idea:

- Graph $\mathbf{G}_{\mathrm{C}} = (\mathbf{V}, \mathbf{C^0} \cup \mathbf{C}^*)$
- Contains edges between all vertex pairs that can be connected **without** the use of **relays**
- Each edge $b \in C^0 \cup C^*$ **corresponds to a set of paths** P_b in G



(a) Original Graph (b) Final Communication Graph

Communication Graph: Example



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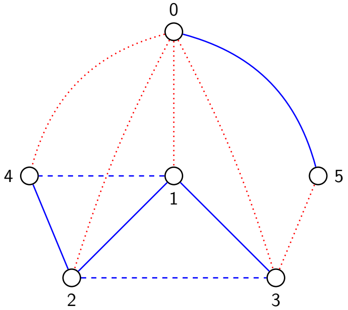
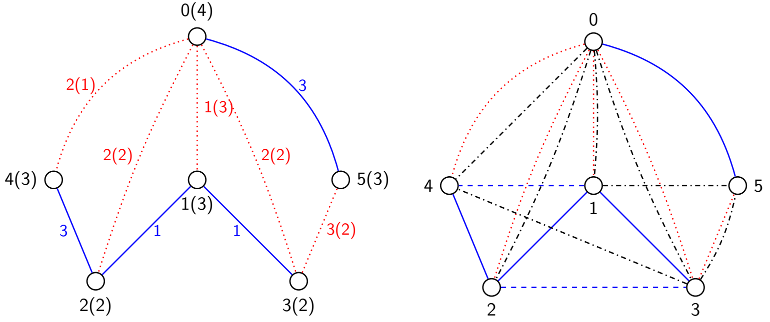


(b) Adding all feasible connections using edges in E_0 (dashed lines)

$$d_{\max} = 4$$

Communication Graph: Example

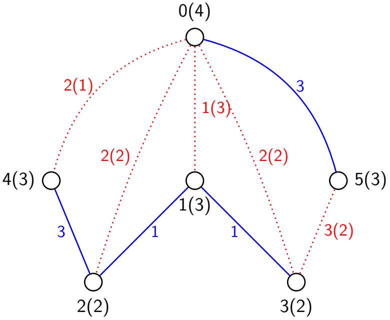


(a) Original Graph

(b) Adding all feasible connections using edges in $E_O \cup E^*$ (dash-dotted lines)

$$d_{\max} = 4$$

Communication Graph: Example



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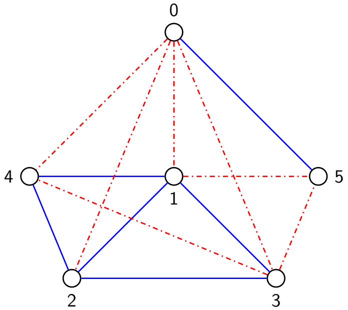


(b) Final Communication Graph

$$d_{\max} = 4$$

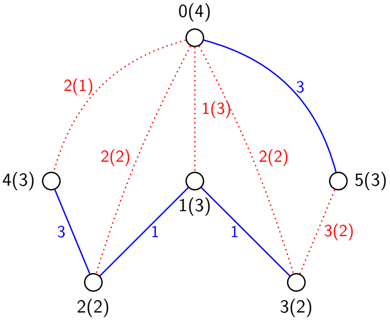
$$G_C = (V, C) \text{ where } C = C^0 \cup C^*$$

Communication Graph: Properties

Consider a pair $(u, v) \in \mathcal{K}$.

- Either u and v are directly connected in G_C, or
- Relays need to be placed, so that there is a u-v path **in $\mathbf{G_C}$** with relay placed at **all intermediate nodes**

Identify cost-optimal connections among the exponentially many possibilities $\Rightarrow$ Column Generation



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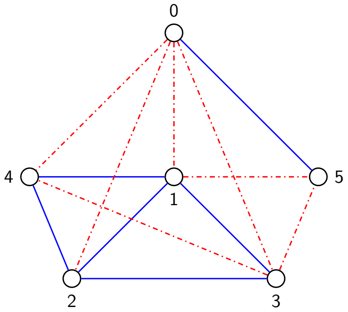
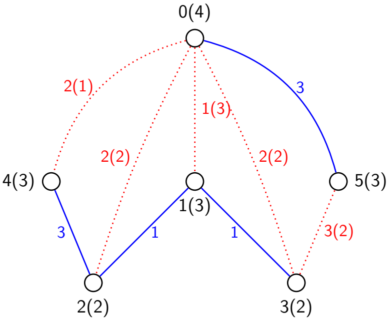


(b) Final Communication Graph

Communication Graph: Properties

Consider a pair $(u, v) = (4, 5)$.

- Place relay at node 1 and use path $(4, 1, 5)$
- Edge $(1, 5) \in C$, so we should choose an appropriate 1-5 path **in G**.
- Since $(1, 5) \in C^*$, edge costs are involved
- Optimal path chosen by Column Generation



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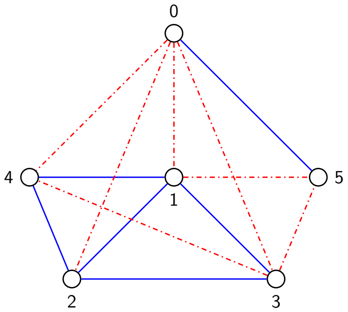

(b) Final Communication Graph

Outline

Communication Graph Models

Theorem

In an optimal solution mapped on a communication graph there exists for every $u \in \mathcal{K}_{\mathrm{S}}$ an arborescence rooted at u reaching all targets $v \in K(u)$.

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Three MIP Formulations

Model Name	Graphs	Connectivity	Type
MCF	comm.graph	multi-commodity flow	B&P
SCF	comm.graph	single-commodity flow	B&P
CUT	directed comm. graph	cutset inequalities	B&P&C

Communication Graph Models

Theorem

In an optimal solution mapped on a communication graph there exists for every $u \in \mathcal{K}_S$ an arborescence rooted at u reaching all targets $v \in K(u)$.

Three MIP Formulations

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CUT	directed comm. graph	cutset inequalities	B&P&C

Major Difficulty

Connect the arborescences in the communication graph with the edges of the original graph.

MCF Variables

$$x_e = \begin{cases} 1, & e \text{ is in solution}, \\ 0, & \text{otherwise} \end{cases} \quad \forall e \in E$$

$$y_i = \begin{cases} 1, & \text{relay is placed at } i, \\ 0, & \text{otherwise} \end{cases} \quad \forall i \in V$$

$$f_a^{uv} = \begin{cases} 1, & a \text{ is used to connect } (u, v) \text{ in } G_{\mathrm{C}}, \\ 0, & \text{otherwise} \end{cases} \quad \forall (u, v) \in \mathcal{K}, \forall a \in A_{\mathrm{C}}$$

Path Variables

Consider a vertex pair $b = (u, v)$. Let

$$P_b = \{p \mid p \text{ is a } u\text{-}v \text{ path in } G, \text{ s.t. } d(p) \leq d_{max}\}$$

Basic idea:

represent each feasible path $p \in P_b$ for a given commodity pair $b = (u, v) \in \mathcal{K}$ as a simple path from u to v in G_{C}.

Path Variables

Mapping between edges in G_{C} and paths in G:

$$\lambda_b^p = \begin{cases} 1, & p \text{ is used to connect node pair } b, \\ 0, & \text{otherwise} \end{cases} \quad \forall b \in C^*, \; \forall p \in P_b$$

Exponential number of λ variables $\Rightarrow$ column generation.

MCF Model

$$\min \sum_{i \in V} c_i y_i + \sum_{e \in E^*} w_e x_e$$

$$\sum_{a \in \delta^-(i)} f_a^{uv} - \sum_{a \in \delta^+(i)} f_a^{uv} = \begin{cases} -1 & i = u \\ 0 & i \neq u, i \neq v \\ 1 & i = v \end{cases} \qquad \forall (u,v) \in \mathcal{K}, \; \forall i \in V_{\mathrm{C}}^u \tag{1}$$

$$\sum_{a \in \delta^+(i)} f_a^{uv} \leq y_i \qquad \forall (u,v) \in \mathcal{K}, \; \forall i \in V, \; i \neq u, i \neq v \tag{2}$$

MCF Model

$$\min \sum_{i \in V} c_i y_i + \sum_{e \in E^*} w_e x_e$$

$$\sum_{a \in \delta^-(i)} f_a^{uv} - \sum_{a \in \delta^+(i)} f_a^{uv} = \begin{cases} -1 & i = u \\ 0 & i \neq u, i \neq v \\ 1 & i = v \end{cases} \qquad \forall (u,v) \in \mathcal{K}, \ \forall i \in V_{\mathrm{C}}^u \qquad (1)$$

$$\sum_{a \in \delta^+(i)} f_a^{uv} \leq y_i \qquad \forall (u,v) \in \mathcal{K}, \ \forall i \in V, \ i \neq u, i \neq v \qquad (2)$$

$$f_{ij}^{uv} + f_{ji}^{uv} \leq \sum_{p \in P_b} \lambda_b^p \qquad \forall (u,v) \in \mathcal{K}, \ \forall b = \{i,j\} \in C^* \quad (\mu_b^{uv}) \qquad (3)$$

$$\sum_{p \in P_b : e \in p} \lambda_b^p \leq x_e \qquad \forall e \in E^*, \forall b \in C^* \quad (\alpha_b^e) \qquad (4)$$

MCF: Pricing Subproblem

Given current LP solution, let $\tilde{\mu}_b^{uv}$ and $\tilde{\alpha}_b^e$ be values of dual variables.

Pricing Subproblem for each $b = \{i, j\} \in C^*$

Find a feasible path $p \in P_b$ with minimum reduced costs:

$$\underset{p \in P_b}{\arg\min} \left(\sum_{e \in E^* \cap p} \tilde{\alpha}_b^e - \sum_{(u,v) \in \mathcal{K}} \tilde{\mu}_b^{uv} \right)$$

For a fixed b, $\sum_{(u,v) \in \mathcal{K}} \tilde{\mu}_b^{uv}$ is a constant, the problem boils down to:

$$\tilde{p}_b = \underset{p \in P_b}{\arg\min} \sum_{e \in E^* \cap p} \tilde{\alpha}_b^e \qquad \forall b \in C^*$$

Weight Constrained Shortest Path Problem. Pseudo-polynomial time.
Dynamic programming procedure proposed by Gouveia et al (2008).

MCF: Pros and Cons

Cons:
Huge number of design variables ($|V| + |E| + |\mathcal{K}||V|^2$) (in addition to λ!)

Pros:
Strong LBs!

Trade the size of the model and the strength of the bounds:
single-commodity flow model *SCF*
(not shown in this talk)

CUT Formulation: Directed Communication Graph

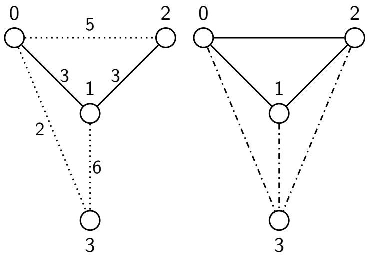
Basic idea:
A feasible path p for a given commodity pair $b = (u, v) \in \mathcal{K}$ is a directed path from u to v in G'_{C}.

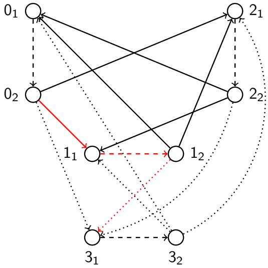
$$x_e = \begin{cases} 1, & e \text{ is in solution,} \\ 0, & \text{otherwise} \end{cases} \forall e \in E$$

$$X_a = \begin{cases} 1, & a \text{ is used in a feasible path,} \\ 0, & \text{otherwise} \end{cases} \forall a \in A'_{\mathrm{C}}$$

One-to-one correspondence between the arcs in A^r_{C} and the relays.

Example: a path between 0 and 3 in communication using a relay at 1





CUT

$$\min \sum_{i \in V} c_i X_{(i_1, i_2)} + \sum_{e \in E*} w_e x_e$$

CUT

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$$\sum_{a \in \delta^-(W)} X_a \geq 1 \qquad \begin{array}{r} \forall (u,v) \in \mathcal{K}, \forall W \subset V'_{\mathrm{C}} \\ v_1 \in W, u_2 \notin W \end{array}$$

CUT

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$$\sum_{a \in \delta^-(W)} X_a \geq 1 \qquad \forall (u,v) \in \mathcal{K}, \forall W \subset V'_{\mathrm{C}}$$
$$v_1 \in W, u_2 \notin W$$

$$X_{uv} \leq \sum_{p \in P_b} \lambda_b^p \qquad \forall b = \{u,v\} \in C^* \quad (\mu_b^1)$$

$$X_{vu} \leq \sum_{p \in P_b} \lambda_b^p \qquad \forall b = \{u,v\} \in C^* \quad (\mu_b^2)$$

CUT

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$$\sum_{p \in P_b : e \in p} \lambda_b^p \leq x_e \qquad \forall e \in E^*, \forall b \in C^* \quad (\alpha_b^e)$$

$$X_a, x_e \in \{0,1\} \qquad \forall e \in E^*, \forall a \in A'_{\mathrm{C}}$$

$$\lambda_b^p \geq 0 \qquad \forall b \in C^*, p \in P_b$$

CUTS: Pricing Subproblem

Pricing Subproblem for each $b \in C^*$

Find a feasible path $p \in P_b$ with minimum reduced costs:

$$\arg\min_{p \in P_b} \left(\sum_{e \in E^* \cap p} \tilde{\alpha}_b^e - \tilde{\mu}_b^1 - \tilde{\mu}_b^2 \right)$$

For a fixed b, $\tilde{\mu}_b^1 + \tilde{\mu}_b^2$ is a constant, the problem boils down to:

$$\tilde{p}_b = \arg\min_{p \in P_b} \sum_{e \in E^* \cap p} \tilde{\alpha}_b^e \qquad \forall b \in C^*$$

Weight Constrained Shortest Path Problem. **Advantage:** connectivity cuts do not influence the pricing subproblem. Row- and column-generation can be performed independently!!

Outline

Cabral - Instances

- Originally introduced in Cabral et al. [2007]
- 4-grid graphs
- Only augmenting edges
- Cost and edge length values are selected uniformly at random from $[10, 30]$
- Communication pairs have a common origin node

Instance	$\|V\|$	$\|E^*\|$	$\|E^0\|$	$\|\mathcal{K}\|$	d_{max}
4A5B70L5K	20	31	0	5	70
4A5B70L10K	20	31		10	
5A5B70L5K	25	40	0	5	70
5A5B70L10K	25	40		10	
6A5B70L5K	30	49	0	5	70
6A5B70L10K	30	49		10	
7A5B70L5K	35	58	0	5	70
7A5B70L10K	35	58		10	
8A5B70L5K	40	67	0	5	70
8A5B70L10K	40	67		10	
9A5B70L5K	45	76	0	5	70
9A5B70L10K	45	76		10	
10A5B70L5K	50	85	0	5	70
10A5B70L10K	50	85		10	
11A5B70L5K	55	94	0	5	70
11A5B70L10K	55	94		10	
12A5B70L5K	60	103	0	5	70
12A5B70L10K	60	103		10	

Results: Cabral - Instances[1]

Instance	LP Gap [%]			LP - t [s]			Optimality Gap [%]			t [s]			RMPI
	MCFM	*SCFM*	*CUTS*	*MCFM*	*SCFM*	*CUTS*	*MCFM*	*SCFM*	*CUTS*	*MCFM*	*SCFM*	*CUTS*	
4A5B70L5K	**18.8**	69.0	23.7	**0**	**0**	**0**	**0.0**	**0.0**	**0.0**	**1**	**1**	**1**	1
4A5B70L10K	**36.6**	79.5	42.2	**0**	**0**	**0**	**0.0**	**0.0**	**0.0**	**5**	17	7	4
5A5B70L5K	**28.6**	67.9	33.8	**0**	**0**	**0**	**0.0**	**0.0**	**0.0**	**2**	9	5	2
5A5B70L10K	**39.3**	81.2	46.8	1	**0**	**0**	**0.0**	**0.0**	**0.0**	**18**	196	25	30
6A5B70L5K	**27.9**	67.0	32.9	**0**	**0**	**0**	**0.0**	**0.0**	**0.0**	**6**	30	10	8
6A5B70L10K	**40.3**	78.6	46.3	2	**0**	1	**0.0**	**0.0**	**0.0**	42	433	**40**	88
7A5B70L5K	**32.8**	66.4	38.7	**0**	**0**	**0**	**0.0**	**0.0**	**0.0**	**8**	21	11	7
7A5B70L10K	**39.7**	78.5	45.1	4	**0**	1	**0.0**	**0.0**	**0.0**	102	1312	**89**	184
8A5B70L5K	**31.8**	69.2	36.0	1	**0**	**0**	**0.0**	**0.0**	**0.0**	**21**	238	32	34
8A5B70L10K	**38.5**	79.9	43.1	5	**0**	1	**0.0**	1.9	**0.0**	**130**	3772	157	523
9A5B70L5K	**31.8**	67.7	38.5	1	**0**	**0**	**0.0**	**0.0**	**0.0**	**24**	257	36	21
9A5B70L10K	**39.0**	81.7	42.5	8	**0**	2	**0.0**	10.8	**0.0**	282	5303	**271**	339
10A5B70L5K	**31.2**	65.9	34.8	2	**0**	**0**	**0.0**	1.3	**0.0**	**59**	1068	81	46
10A5B70L10K	**38.1**	77.4	42.7	13	**0**	1	**0.0**	12.4	**0.0**	844	5674	**547**	1419
11A5B70L5K	**34.0**	67.0	39.1	2	**0**	**0**	**0.0**	**0.0**	**0.0**	**40**	458	50	34
11A5B70L10K	**38.0**	76.7	42.4	22	**0**	2	**0.0**	18.3	**0.0**	1073	7056	**588**	3496
12A5B70L5K	**34.9**	68.0	39.1	3	**0**	1	**0.0**	2.3	**0.0**	**459**	1085	660	549
12A5B70L10K	**38.0**	78.3	42.2	27	**0**	2	4.1	25.0	**1.3**	2661	6587	**2196**	4656

[1]Each row corresponds to the mean over 10 instances.

RMPI: Cabral et al. (2007) report gaps between 3% and 21%.

NDPR - Instances

- Modified version of the instances provided in Konak [2012]
- Euclidean distances have been rounded up to obtain integral edge weights and lengths
- Only augmenting edges
- Type I: $w_e = d_e$
- Type II: $w_e = d_{max} - d_e$

Instance	$\|V\|$	$\|E^*\|$	$\|E^0\|$	$\|\mathcal{K}\|$	d_{max}
40N5K30L	40	198	0	5	30
40N5K35L		272		5	35
40N10K30L		198		10	30
40N10K35L		272		10	35
50N5K30L	50	279	0	5	30
50N5K35L		372		5	35
50N10K30L		279		10	30
50N10K35L		372		10	35
60N5K30L	60	305	0	5	30
60N5K35L		412		5	35
60N10K30L		305		10	30
60N10K35L		412		10	35
80N5K30L	80	641	0	5	30
80N5K35L		853		5	35
80N10K30L		641		10	30
80N10K35L		853		10	35
160N5K30L	160	2773	0	5	30
160N5K35L		3624		5	35
160N10K30L		2773		10	30
160N10K35L		3624		10	35

Results: NDPR - Instances (Type I)

Instance	LP Gap [%]			LP - t [s]			Optimality Gap [%]			t [s]		
	MCFM	*SCFM*	*CUTS*	*MCFM*	*SCFM*	*CUTS*	*MCFM*	*SCFM*	*CUTS*	*MCFM*	*SCFM*	*CUTS*
40N_5K_30L	**21.5**	**21.5**	24.9	**0**	**0**	**0**	**0.0**	**0.0**	**0.0**	76	**72**	98
40N_5K_35L	**38.5**	**38.5**	43.4	3	3	**1**	11.6	**11.4**	11.8	7200	7200	7200
40N_10K_30L	**26.2**	26.9	28.9	4	**3**	**0**	**0.0**	**0.0**	**0.0**	2850	3109	**1163**
40N_10K_35L	**40.1**	40.3	44.6	6	5	**2**	**17.5**	**17.5**	20.8	7200	7200	7200
50N_5K_30L	**8.5**	13.5	15.5	**1**	**1**	**1**	**0.0**	**0.0**	**0.0**	**6**	8	20
50N_5K_35L	**22.5**	25.4	27.6	**2**	**2**	**2**	**0.0**	**0.0**	**0.0**	**39**	46	83
50N_10K_30L	**24.6**	25.2	31.1	9	**7**	**2**	**0.0**	**0.0**	**0.0**	**737**	828	1841
50N_10K_35L	**27.6**	28.1	37.4	22	**18**	**6**	**0.0**	**0.0**	**0.0**	2098	**1890**	3209
60N_5K_30L	**28.1**	**28.1**	35.8	**2**	**2**	**2**	**0.0**	**0.0**	**0.0**	**63**	67	236
60N_5K_35L	**24.3**	**24.3**	34.1	**4**	**5**	**4**	**0.0**	**0.0**	**0.0**	141	**121**	429
60N_10K_30L	**32.3**	**32.3**	40.3	11	**11**	**2**	**0.0**	**0.0**	5.0	**2534**	2908	7200
60N_10K_35L	**33.1**	**33.1**	43.7	17	**36**	**6**	**0.0**	3.6	12.8	**7155**	7200	7200
80N_5K_30L	**44.6**	51.0	52.2	24	16	**14**	**28.6**	34.4	33.9	7200	7200	7200
80N_5K_35L	**53.7**	59.2	61.6	83	49	**33**	**41.8**	47.9	45.0	7200	7200	7200
80N_10K_30L	**53.1**	56.5	59.9	195	139	**26**	**44.0**	46.4	47.4	7200	7200	7200
80N_10K_35L	**66.3**	68.3	73.1	1046	602	**63**	**63.7**	65.1	64.2	7200	7200	7200
160N_5K_30L	**55.8**	**55.8**	64.5	4183	3625	**1414**	55.8	**54.9**	62.9	7200	7200	7200
160N_5K_35L	-	-	**73.4**	7200	7200	**6402**	-	-	**73.2**	7200	7200	7200
160N_10K_30L	-	-	**74.5**	7200	7200	7200	-	-	**74.5**	7200	7200	7200
160N_10K_35L	-	-	**85.8**	7200	7200	7200	-	-	**85.8**	7200	7200	7200

Results: NDPR - Instances (Type II)

Instance	LP Gap [%]			LP - t [s]			Optimality Gap [%]			t [s]		
	MCFM	*SCFM*	*CUTS*	*MCFM*	*SCFM*	*CUTS*	*MCFM*	*SCFM*	*CUTS*	*MCFM*	*SCFM*	*CUTS*
40N_5K_30L_p3	**0.3**	**0.3**	1.6	**0**	**0**	**0**	**0.0**	**0.0**	**0.0**	**0**	**0**	**0**
40N_5K_35L_p3	**7.8**	**7.8**	12.9	2	**1**	**1**	**0.0**	**0.0**	**0.0**	6	**4**	6
40N_10K_30L_p3	**6.6**	8.0	9.7	2	**2**	**0**	**0.0**	**0.0**	**0.0**	11	10	**9**
40N_10K_35L_p3	**10.1**	10.9	16.1	7	**7**	**1**	**0.0**	**0.0**	**0.0**	61	55	**29**
50N_5K_30L_p3	**0.0**	6.5	**0.0**	**0**	**0**	**0**	**0.0**	**0.0**	**0.0**	**0**	1	**0**
50N_5K_35L_p3	**1.9**	9.8	7.4	**1**	**1**	2	**0.0**	**0.0**	**0.0**	**2**	4	22
50N_10K_30L_p3	**2.5**	5.7	7.6	**1**	**1**	2	**0.0**	**0.0**	**0.0**	3	**2**	21
50N_10K_35L_p3	**3.0**	4.0	11.4	**7**	**8**	**7**	**0.0**	**0.0**	**0.0**	56	**39**	64
60N_5K_30L_p3	**14.9**	**14.9**	23.0	**1**	**2**	**1**	**0.0**	**0.0**	**0.0**	**8**	**8**	27
60N_5K_35L_p3	**0.0**	**0.0**	**0.0**	**0**	**0**	2	**0.0**	**0.0**	**0.0**	**0**	**0**	2
60N_10K_30L_p3	**13.7**	**13.7**	19.6	4	**5**	**2**	**0.0**	**0.0**	**0.0**	58	53	**44**
60N_10K_35L_p3	**6.9**	**6.9**	10.2	6	**7**	**4**	**0.0**	**0.0**	**0.0**	39	**30**	39
80N_5K_30L_p3	**7.8**	15.9	13.3	**8**	**6**	11	**0.0**	**0.0**	**0.0**	**54**	85	117
80N_5K_35L_p3	**6.2**	16.1	12.0	**13**	**14**	36	**0.0**	**0.0**	**0.0**	**59**	208	229
80N_10K_30L_p3	**12.1**	16.1	18.4	86	**55**	**22**	**0.0**	**0.0**	**0.0**	1227	1105	**526**
80N_10K_35L_p3	**18.9**	23.4	25.6	348	244	**118**	14.5	15.0	**8.6**	7200	7200	7200
160N_5K_30L_p3	**7.3**	**7.3**	11.6	**715**	**658**	882	**0.0**	**0.0**	**0.0**	**1896**	1954	6641
160N_5K_35L_p3	**10.9**	**10.9**	14.1	**1301**	1348	2119	**5.1**	5.7	9.5	7200	7200	7200
160N_10K_30L_p3	**14.1**	**14.1**	19.4	**2553**	2843	2584	**13.8**	**13.8**	17.8	7200	7200	7200
160N_10K_35L_p3	-	-	**33.2**	7200	7200	7200	-	-	**33.2**	7200	7200	7200

ARLP - Instances

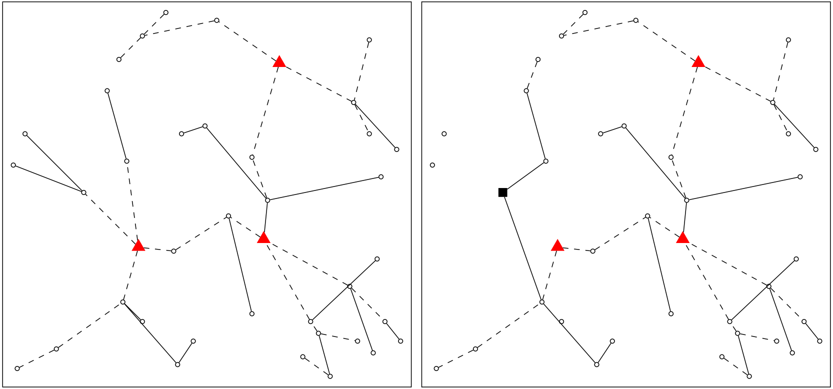
- New Instances with a high amount of commodities
- Edge lengths based on rounded up euclidean distances
- Edge costs normally distributed around edge lengths ($\mu = d_e, \sigma = 5$)
- Also free edges

Instance	$\|V\|$	$\|E^*\|$	$\|E^0\|$	$\|\mathcal{K}\|$	d_{max}
40N50L20F_A	40	124	26	724	50
40N50L20F_B		123	35	688	
40N50L50F_A		78	89	513	
40N50L50F_B		72	71	586	
40N50L80F_A		32	146	443	
40N50L80F_B		35	154	423	
50N50L20F_A	50	212	44	1111	50
50N50L20F_B		235	59	1022	
50N50L50F_A		157	132	719	
50N50L50F_B		132	117	873	
50N50L80F_A		51	175	788	
50N50L80F_B		58	212	682	
60N50L20F_A	60	269	72	1549	50
60N50L20F_B		268	63	1588	
60N50L50F_A		204	216	1036	
60N50L50F_B		200	197	1103	
60N50L80F_A		85	311	854	
60N50L80F_B		74	283	1041	
80N50L20F_A	80	557	145	2599	50
80N50L20F_B		545	124	2659	
80N50L50F_A		345	313	1922	
80N50L50F_B		375	366	1902	
80N50L80F_A		148	548	1834	
80N50L80F_B		121	536	1709	

Results: ARLP - Instances

Instance	LP Gap [%]			LP - t [s]			Optimality Gap [%]			t [s]		
	MCFM	*SCFM*	*CUTS*	*MCFM*	*SCFM*	*CUTS*	*MCFM*	*SCFM*	*CUTS*	*MCFM*	*SCFM*	*CUTS*
40N50L20F_A	-	31.5	**20.7**	7200	**177**	**4**	-	**0.0**	**0.0**	7200	1517	**189**
40N50L20F_B	-	30.8	**24.3**	7200	**113**	**2**	-	**0.0**	**0.0**	7200	816	**86**
40N50L50F_A	**15.6**	19.6	17.1	157	**4**	**0**	**0.0**	**0.0**	**0.0**	1733	57	**4**
40N50L50F_B	**6.6**	13.2	9.0	471	**5**	**0**	**0.0**	**0.0**	**0.0**	3532	31	**4**
40N50L80F_A	**0.0**	**0.0**	**0.0**	7	**0**	**2**	**0.0**	**0.0**	**0.0**	7	**0**	2
40N50L80F_B	**6.6**	8.7	10.3	23	**1**	**0**	**0.0**	**0.0**	**0.0**	83	10	**2**
50N50L20F_A	ML	26.3	**13.7**	ML	**746**	**8**	ML	14.6	**0.0**	ML	7200	**1268**
50N50L20F_B	ML	26.1	**9.5**	ML	**570**	**7**	ML	9.1	**0.0**	ML	7200	**110**
50N50L50F_A	ML	**14.7**	16.3	ML	**17**	**1**	ML	**0.0**	**0.0**	ML	68	**6**
50N50L50F_B	ML	9.8	**7.4**	ML	**21**	**1**	ML	**0.0**	**0.0**	ML	192	**22**
50N50L80F_A	ML	**14.1**	17.1	ML	**2**	**0**	ML	**0.0**	**0.0**	ML	10	**5**
50N50L80F_B	ML	0.3	**0.0**	ML	**1**	**4**	ML	**0.0**	**0.0**	ML	**2**	4
60N50L20F_A	ML	25.8	**13.6**	ML	**1193**	**15**	ML	16.4	**0.0**	ML	7200	**862**
60N50L20F_B	ML	37.4	**25.3**	ML	**1401**	**20**	ML	33.1	**0.0**	ML	7200	**4158**
60N50L50F_A	ML	**14.3**	15.0	ML	**49**	**1**	ML	**0.0**	**0.0**	ML	436	**21**
60N50L50F_B	ML	20.4	**10.4**	ML	**60**	**1**	ML	**0.0**	**0.0**	ML	787	**25**
60N50L80F_A	ML	**2.2**	**2.2**	ML	**12**	**1**	ML	**0.0**	**0.0**	ML	27	**9**
60N50L80F_B	ML	**12.1**	12.7	ML	**7**	**1**	ML	**0.0**	**0.0**	ML	27	**10**
80N50L20F_A	ML	-	**26.0**	ML	7200	**89**	ML	-	**20.5**	ML	7200	7200
80N50L20F_B	ML	-	**31.9**	ML	7200	**94**	ML	-	**30.9**	ML	7200	7200
80N50L50F_A	ML	21.2	**18.2**	ML	**438**	**3**	ML	**0.0**	**0.0**	ML	6054	**104**
80N50L50F_B	ML	**11.7**	16.2	ML	**497**	**3**	ML	**0.0**	**0.0**	ML	3807	**56**
80N50L80F_A	ML	**2.6**	3.2	ML	**49**	**4**	ML	**0.0**	**0.0**	ML	107	**29**
80N50L80F_B	ML	**4.2**	5.8	ML	**57**	**2**	ML	**0.0**	**0.0**	ML	223	**31**

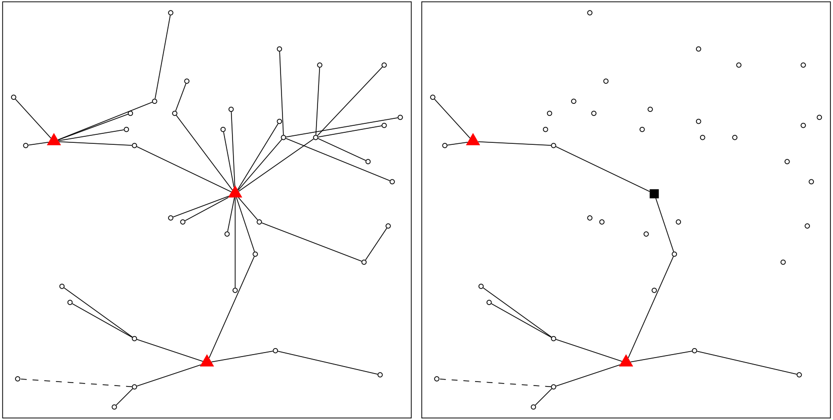
Example 1



(a) 40N30C50L20F_A (original) (b) 40N30C50L20F_A ($\mathcal{K}'$)

Solid lines: free edges, dashed lines: augmenting edges.
Selected relays: triangles. Single source: square.
724 commodities (left).

Example 2



(a) 40N30C50L80F_A (original) (b) 40N30C50L80F_A ($\mathcal{K}'$)

80% of free edges. Solid lines: free edges, dashed lines: augmenting edges.
Selected relays: triangles. Single source: square.
443 commodities (left).

Conclusion

Summary:

- First study on exact algorithms for NDPR
- MIP models based on communication graph(s)
- Flow-based models (B&P) work well if <u>very few commodities</u> involved
- Cut-based model (B&P&C) better for a <u>large number</u> of commodities
- The new algorithms find optimal solutions for instances of:
  - 160 nodes and more than 3500 edges
  - 80 nodes and more than 1900 commodity pairs

Conclusion

Future Challenges:

- Not included in our model:
  - maximal number of charging stations along a single trip
  - maximal detouring length
  - maximal duration of the trip
  - different charging technologies

Conclusion

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- Not included in our model:
  - maximal number of charging stations along a single trip
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  - maximal duration of the trip
  - different charging technologies
- Still, the obtained solutions:
  - can be used within some decomposition schemes (Lagrangian, Benders)
  - in a step-by-step planning approach
  - as a starting heuristic solution that needs small repairs

Conclusion

Future Challenges:

- Not included in our model:
  - maximal number of charging stations along a single trip
  - maximal detouring length
  - maximal duration of the trip
  - different charging technologies
- Still, the obtained solutions:
  - can be used within some decomposition schemes (Lagrangian, Benders)
  - in a step-by-step planning approach
  - as a starting heuristic solution that needs small repairs
- More powerful exact algorithms, matheuristics, ...
- Models on layered graphs

Thank you for your Attention! Questions?