Short-term Unit Commitment Problem in Hydro Valleys: Overview and Reformulations

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Joint work with:

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Outline

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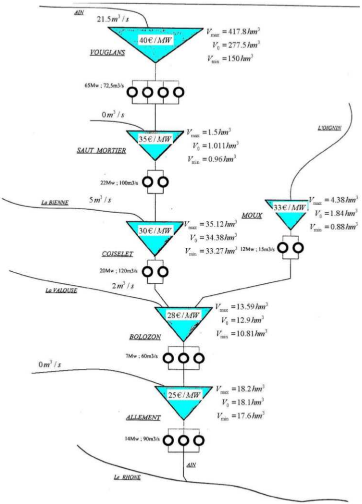
Why Hydraulic Energy ?

"More than 95% of Norwegian production comes from hydro plants."

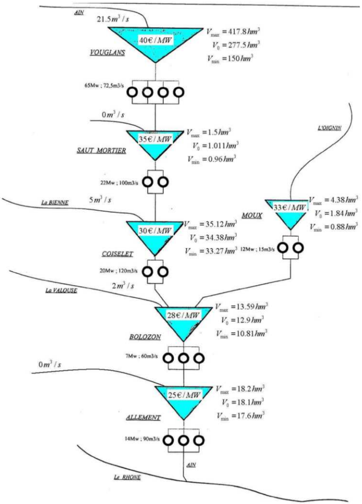
Stein-Erik Fleten et Stein W Wallace. "Delta-hedging a hydropower plant using stochastic programming". In : *Optimization in the energy industry*. 2009

Hydro Valleys



- Reservoirs : natural/artificial water basins,
- Turbines : produce power by using the water's potential energy,
- Pumps : transfer water back to the reservoir by using some power.

Short-term Hydro Unit Commitment Problem

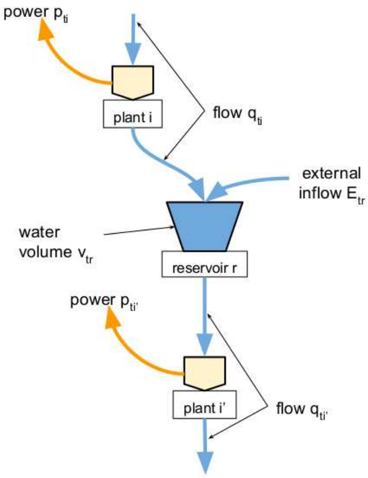


Find an optimal short-term scheduling of several plants with multi-unit-pump-storage hydro power station.

More Precisely ! !

At each time period t :

1. For each reservoir : water volume v_t,
2. For each unit (turbine/pump) :
   - Startup/shutdown, on/off status,
   - Water flow q_{jt} (pump : $q_{jt} < 0$, turbine : $q_{jt} > 0$, not operating : $q_{jt} = 0$),
   - Power generated or consumed p_{jt} (pump : $p_{jt} < 0$, turbine : $p_{jt} > 0$, not operating : $p_{jt} = 0$).



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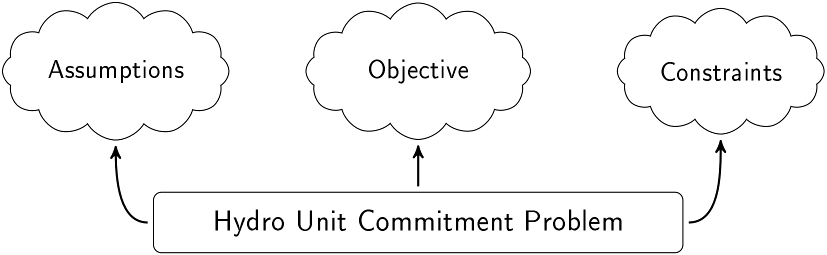
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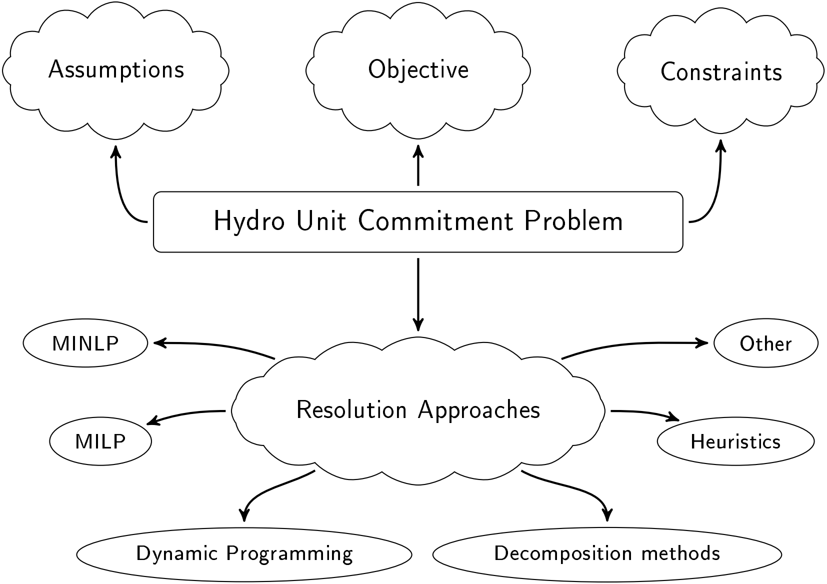
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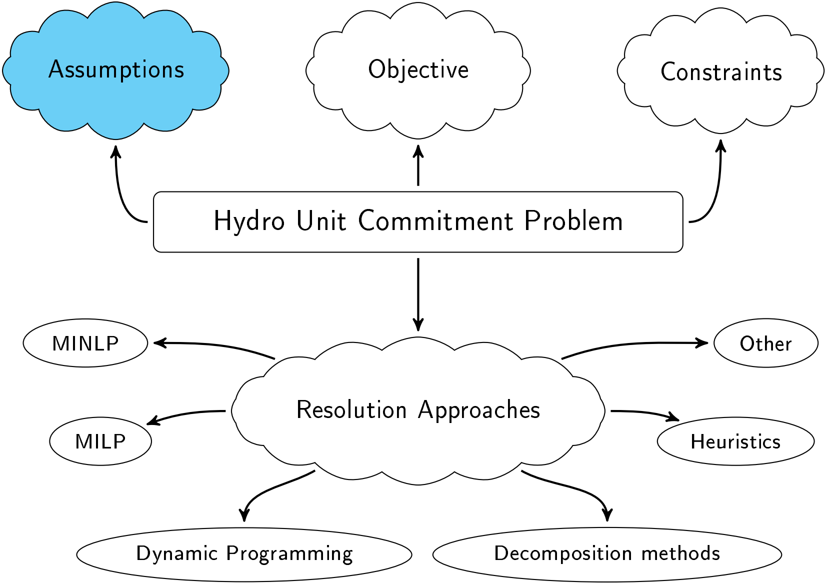
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Assumptions

- Hydro units (turbines,pumps),
- Short-term : a day, a week,
- Deterministic : forecast inflows,
- Price-taker : forecast electricity prices.

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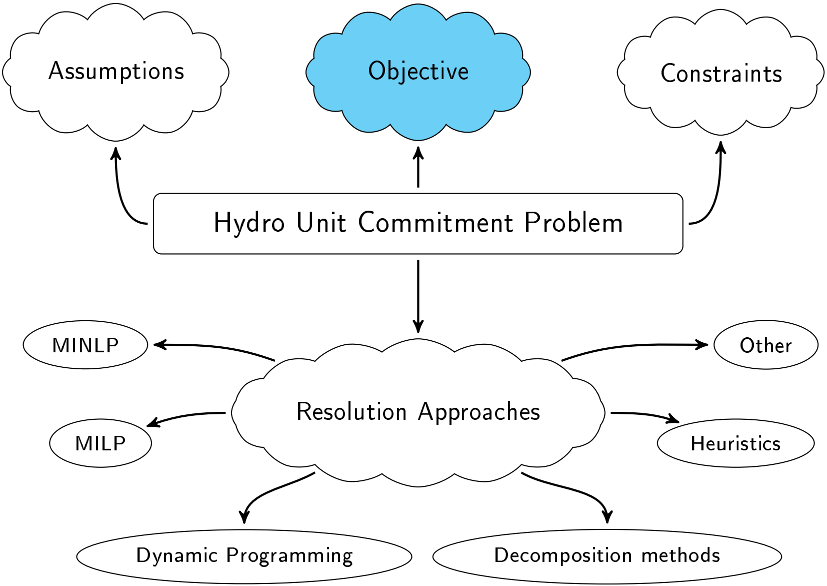
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- Min hydro-logic alteration/damage by inundation
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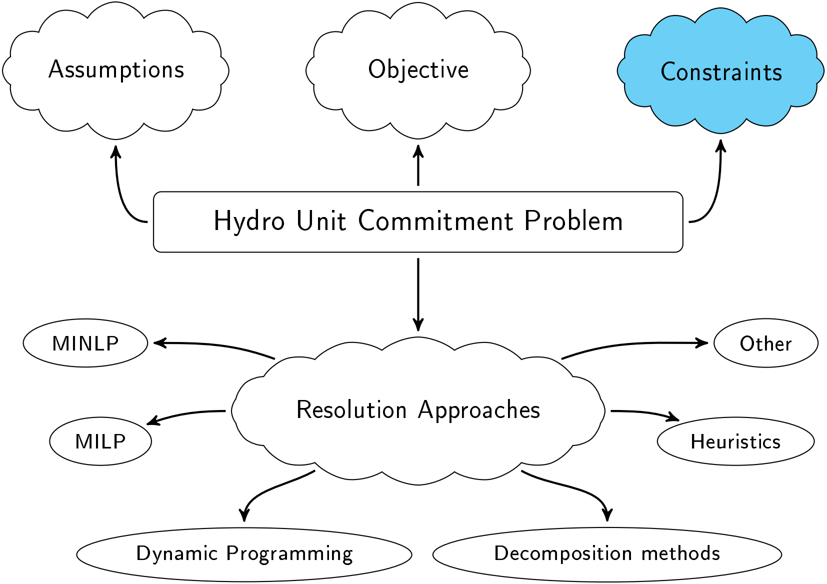
⋆ R. Dittmann et al. "Optimum multi-objective reservoir operation with emphasis on flood control and ecology". In : *Natural Hazards & Earth System Sciences* 9.6 (2009)

⋆ Glauber R Colnago et Paulo B Correia. "Multiobjective dispatch of hydrogenerating units using a two-step genetic algorithm method". In : *Evolutionary Computation, 2009. CEC'09. IEEE Congress on*. IEEE. 2009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Constraints

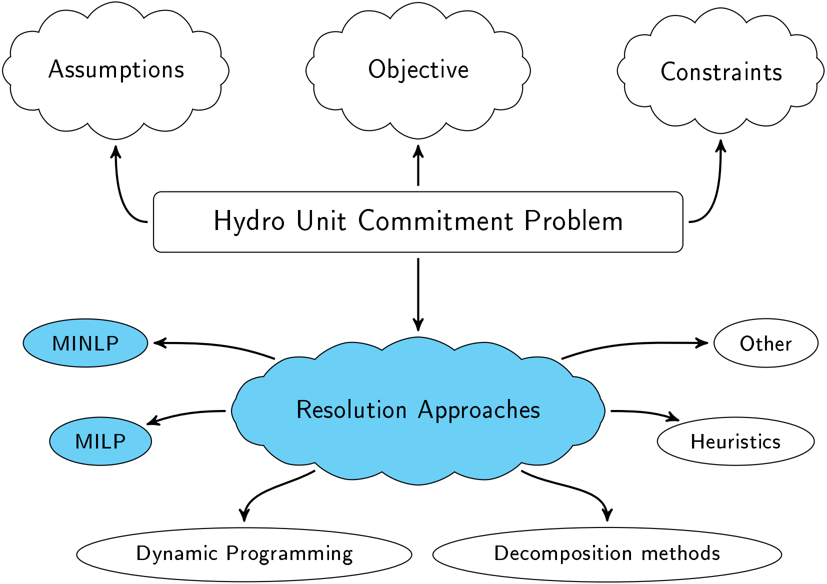
1. Physical Constraints : typically hard constraints
   - Water flow balance equations
   - Forbid of simultaneous pump and turbine mode
   - Minimum number of periods to be spent in a status by the unit
   - Respect allowed operational points
   - Spinning reserves
   - spillage

Constraints

2. Strategic Constraints : typically soft constraints
   - Irrigation requirement, Ecological flows, Water rights
   - Load balance equations constraints
   - Minimum release of water per period
   - Final reservoir level

Challenges

- Large-scale problem
- Should be solved in short amount of time
- Combinatorial aspects
- Non-linearities
- Multiple (conflicting) objectives
- Non-feasibility of some instances
- ...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1. Non-linearity : where ?
   - Focus on MINLP with non-linear objective function and linear constraints.
   - The power production : a highly non-linear function of the water flow and either the water level or (equivalently) the water volume in the reservoir.

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2. Why ?

   Accurate description of the hydropower plants characteristics, head variation effects on the output power
   $\longrightarrow$ more realistic and feasible results

MINLP

- ⋆ JPS Catalão et al. "Parameterisation effect on the behaviour of a head-dependent hydro chain using a nonlinear model". In : *Electric Power Systems Research* 76.6 (2006)
- ⋆ JPS Catalão et al. "Nonlinear optimization method for short-term hydro scheduling considering head-dependency". In : *European Transactions on Electrical Power* 20.2 (2010)
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- ⋆ FJavier Diaz et al. "Optimal scheduling of a price-taker cascaded reservoir system in a pool-based electricity market". In : *Power Systems, IEEE Transactions on* 26.2 (2011)
- ⋆ J Shu et al. "Self-scheduling of cascaded hydropower stations based on Nonlinear Complementarity approach". In : *Power System Technology (POWERCON), 2010 International Conference on*. IEEE. 2010

MILP

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1. Linearity : why ?
   - MILP solvers more efficient than MINLP ones and handle large-scale instances
   - Trying to get rid of the non-linear functions → "linearize" and use MILP solvers

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   - MILP solvers more efficient than MINLP ones and handle large-scale instances
   - Trying to get rid of the non-linear functions $\rightarrow$ "linearize" and use MILP solvers
2. How ?
   - Piecewise linear approximation (easily applied for univariate functions)
   - Functions of 2 variables (fix one of the variables)
   - Triangulation

MILP

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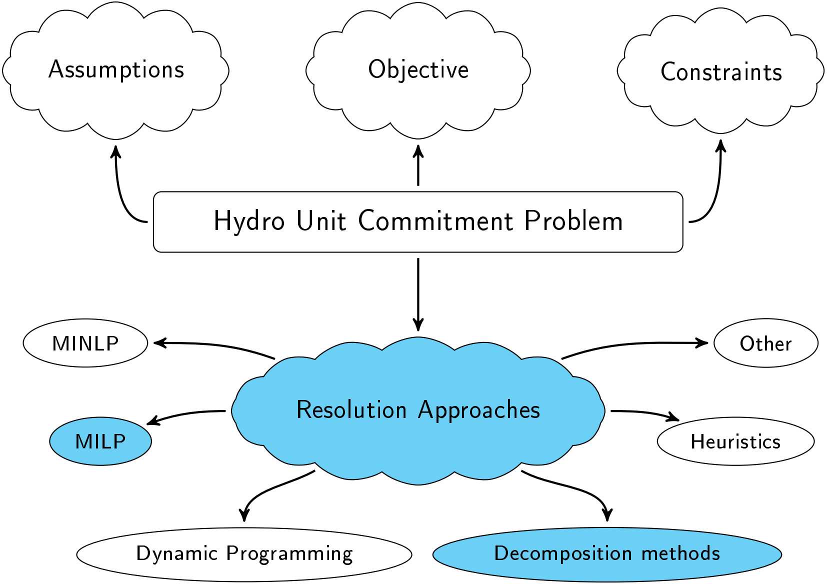
- ⋆ Jiangtao Jia et Xiaohong Guan. "MILP formulation for short-term scheduling of cascaded reservoirs with head effects". In : *Artificial Intelligence, Management Science and Electronic Commerce (AIMSEC), 2011 2nd International Conference on*. IEEE. 2011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Outline

Single Reservoir HUC Problem (Borghetti et al., '08)

1. Sets
   - $T = \{1, \ldots, \overline{t}\}$ = set of time periods
   - $J = \{1, \ldots, \overline{n}\}$ = set of turbine/pump units

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   - p_{jt} : power generated or consumed by unit j in period t $(j \in J, t \in T)$
   - v_t : water volume in the basin in period t $(t \in T)$
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- Other variables for linearization

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   Plus
   - Binary variables for Startups/Shutdowns and pump/turbine status
   - Other variables for linearization
3. **Objective** : Power selling - Turbine/Pump startup cost

$$\max \sum_{j \in J} \sum_{t \in T} \left(\Delta t \, \Pi_t \, p_{jt} - C_j \, \widetilde{w}_{jt} - (D_j + \Pi_t E_j) \widetilde{y}_{jt} \right)$$

Turbine/Pump possible status

For each period t, we have the three possible cases that can occur relative to turbine/pump unit j :

- if unit j is generating power $\rightarrow q_{jt} > 0$ and $p_{jt} > 0$;
- if unit j is pumping water $\rightarrow q_{jt} < 0$ and $p_{jt} < 0$;
- if unit j is not operating $\rightarrow q_{jt} = 0$ and $p_{jt} = 0$.

Reservoirs' constraints ⇒ Global constraints

1. Bounds

$$\underline{V} \leq v_t \leq \overline{V}, \qquad 0 \leq s_t \leq \overline{S} \qquad \forall t \in T$$

2. Final reservoir level

$$v_{\overline{t}} - V_{\overline{t}} \geq 0$$

3. Water volume conservation

$$v_t = v_{t-1} + 3600\,\Delta t\,(I_t - \sum_{j \in J} q_{jt} - s_t) \qquad \forall t \in T$$

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4. Spillage

$$s_t - \sum_{j \in J} (W_j\,\widetilde{w}_{jt} + Y_j\,\widetilde{y}_{jt}) \geq 0 \qquad \forall t \in T$$

$$\sum_{j \in J} q_{jt} + s_t - \underline{\Theta} \geq 0 \quad \forall t \in T$$

Units' constraints $\Rightarrow$ Local constraints

5. Bounds

$$Q_j^- \leq q_{jt} \leq \overline{Q}_j, \quad P_j^- \leq p_{jt} \leq \overline{P}_j, \qquad \forall j \in J, t \in T$$

6. Lower and Upper bounds on turbines flows

$$q_{jt} - (Q_j^- \, u_{jt} + \underline{Q}_j \, g_{jt}) \geq 0, \;\; q_{jt} - (Q_j^- \, u_{jt} + \overline{Q}_j \, g_{jt}) \leq 0 \quad \forall j \in J, t \in T$$

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7. Ramp-up/down

$$\sum_{j \in J}(q_{jt} - q_{j(t-1)}) + \Delta Q^- \geq 0, \; \sum_{j \in J}(q_{jt} - q_{j(t-1)}) - \Delta Q^+ \leq 0 \quad \forall t \in T$$

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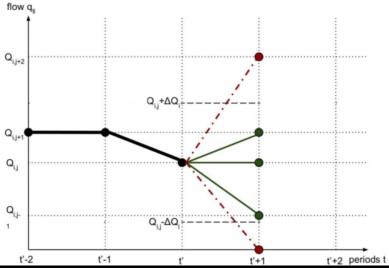

Units' constraints ⇒ Local constraints

8. Switch-on/switch-off for turbines and pumps

$$g_{jt} - g_{j(t-1)} - (\widetilde{w}_{jt} - w_{jt}) = 0, \quad \widetilde{w}_{jt} + w_{jt} \leq 1 \qquad \forall j \in J, t \in T$$

$$u_{jt} - u_{j(t-1)} - (\widetilde{y}_{jt} - y_{jt}) = 0, \quad \widetilde{y}_{jt} + y_{jt} \leq 1 \qquad \forall j \in J, t \in T$$

9. Turbines and pumps status

$$g_{jt} + u_{kt} \leq 1 \, \forall j, k \in J, t \in T,$$

$$\sum_{j \in J} u_{jt} \leq \overline{n} - 1 \qquad \forall t \in T.$$

Outline

In the literature

- Long and mid-term hydro-thermal Unit Commitment Problem,
  - ⋆ JC Enamorado, A Ramos et T Gómez. "Multi-area decentralized optimal hydro-thermal coordination by the Dantzig-Wolfe decomposition method". In : *Power Engineering Society Summer Meeting, 2000. IEEE*. T. 4. IEEE. 2000
  - ⋆ Yong Fu, Mohammad Shahidehpour et Zuyi Li. "Long-term security-constrained unit commitment : hybrid Dantzig-Wolfe decomposition and subgradient approach". In : *Power Systems, IEEE Transactions on* 20.4 (2005)
- Long-term nuclear-thermal Unit Commitment Problem (ROADEF/EURO 2010 Challenge)
  - ⋆ Antoine Rozenknop et al. "Solving the electricity production planning problem by a column generation based heuristic". In : *Journal of Scheduling* 16.6 (2013)

A simplified version !

- Neglect the spillage-related parameters, variables and constraints,
- Do not consider the head effect,
- Relax the target volume constraint and put it as a penalty in the cost function.

Operational points of a unit j

- For each unit j, suppose given a set of operational points $\mathcal{I}$,
- With each operational point are associated possible values of generated or consumed power p_j (water flow q_j),
- Denote p_j^i (q_j^i), $i \in \mathcal{I}$ these values.

$G_j = (N_j, A_j)$: a directed graph associated with unit j

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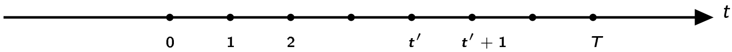

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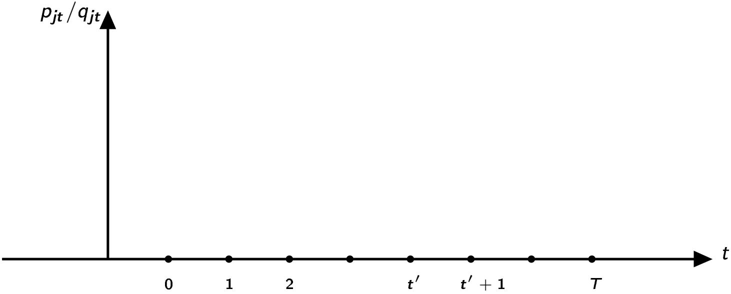

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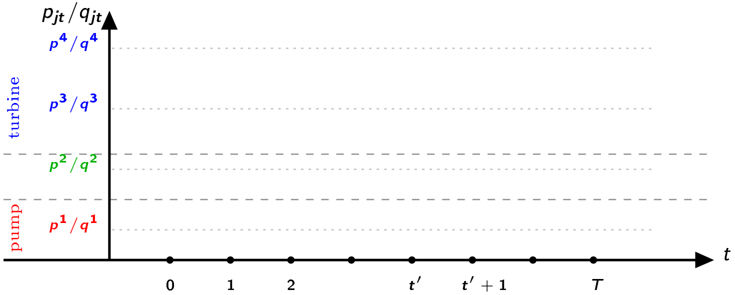

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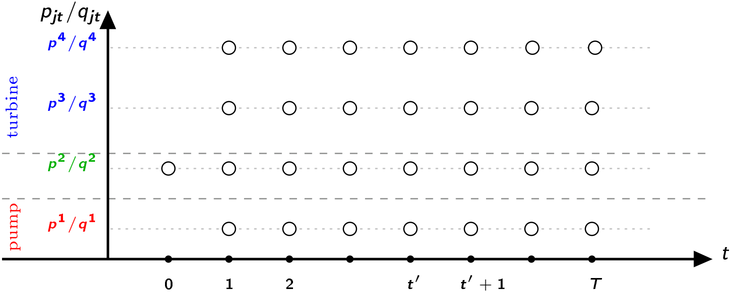

Graph nodes N_j : possible operational points

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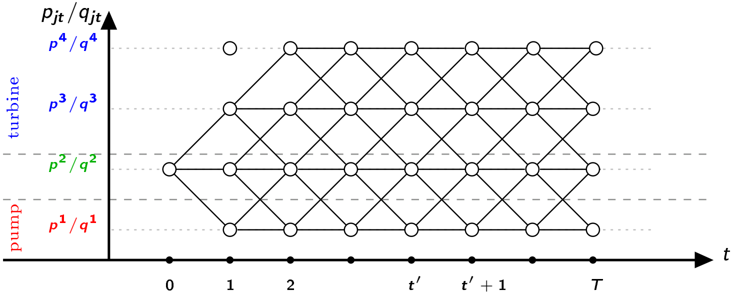
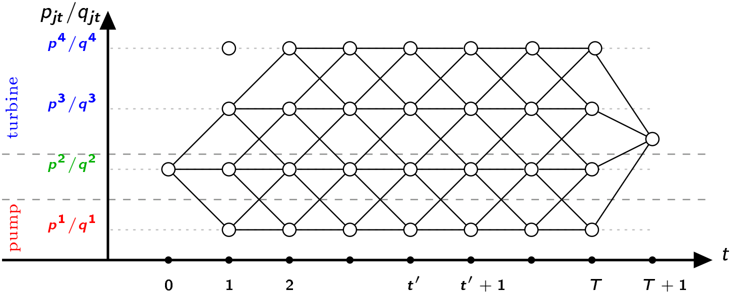


Graph arcs A_j : possible operating status changing

7. Ramp-up/down
$$(q_{jt} - q_{j(t-1)}) + \Delta Q^- \geq 0,\ (q_{jt} - q_{j(t-1)}) - \Delta Q^+ \leq 0 \quad \forall j \in J, t \in T$$
8. Switch-on/switch-off for turbines and pumps
$$g_{jt} - g_{j(t-1)} - (\widetilde{w}_{jt} - w_{jt}) = 0, \quad \widetilde{w}_{jt} + w_{jt} \leq 1 \qquad \forall j \in J, t \in T$$
$$u_{jt} - u_{j(t-1)} - (\widetilde{y}_{jt} - y_{jt}) = 0, \quad \widetilde{y}_{jt} + y_{jt} \leq 1 \qquad \forall j \in J, t \in T$$

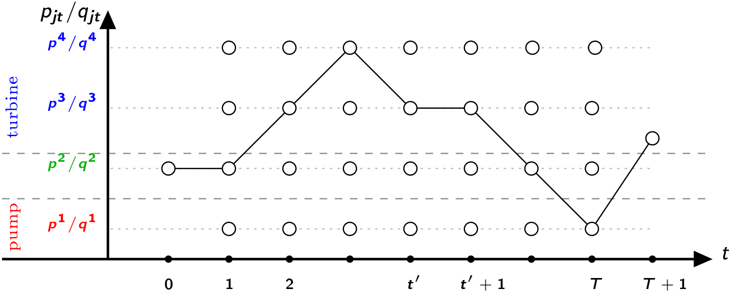
$G_j = (N_j, A_j)$: a directed graph associated with unit j



Graph arcs A_j : possible operating status changing

- fictive node for period $T + 1$ and fictive arcs,
- To each arc $a \in A_j$ are associated two values :
  - a cost c_a corresponding to start-up/shut-down costs minus power selling,
  - a penalty w_a representing water consumption,
  - for fictive arcs $c_a = w_a = 0$.

$G_j = (N_j, A_j)$: a directed graph associated with unit j



Notations

- A path π from 0 to $T + 1$ is the sequence of possible status of unit j
  $\rightarrow$ A possible production plan for unit j
- Denote by $\Pi_j, j \in J$ the set of the possible paths for unit j

Parameters for a unit j

- $b_{j,\pi}(n_t), (\pi \in \Pi_j, n_t \in N_j)_{j \in J}$

$$b(n_t) = \begin{cases} 1 & \text{if } \pi \in \Pi_j \text{ passes through } n_t \\ 0 & \text{otherwise,} \end{cases}$$

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$$b(n_t) = \begin{cases} 1 & \text{if } \pi \in \Pi_j \text{ passes through } n_t \\ 0 & \text{otherwise,} \end{cases}$$

- $q(n_t)$ the flow associated with node $n_t \in N_j$, $j \in J$.

$$q(n_t) = \begin{cases} > 0 & \text{if unit } j \text{ is operating as a turbine at period } t \\ < 0 & \text{if unit } j \text{ is operating as a pump at period } t \\ 0 & \text{if unit } j \text{ is not operating at period } t. \end{cases}$$

Variables for a unit j

Path Variables

$$x_{j,\pi} = \begin{cases} 1 & \text{if } \pi \in \Pi_j \text{ is selected as a plan production for unit j} \\ 0 & \text{otherwise} \end{cases}$$

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Relation with previous variables

- Flow variables (continuous)
  $q_{jt} = \sum_{\pi \in \Pi_j} \sum_{n_t \in N_{jt}} b(n_t) q(n_t) x_{j,\pi} \qquad \forall j \in J, t \in T$
- Turbine status (binary)
  $g_{jt} = \sum_{\pi \in \Pi_j} \sum_{\substack{n_t \in N_{jt}: \\ q(n_t) > 0}} x_{j,\pi} \qquad \forall j \in J, t \in T$
- Pump status (binary)
  $u_{jt} = \sum_{\pi \in \Pi_j} \sum_{\substack{n_t \in N_{jt}: \\ q(n_t) < 0}} x_{j,\pi} \qquad \forall j \in J, t \in T$

Path-based formulation

$$\min \sum_{j \in J} \sum_{\pi \in \Pi_j} c_\pi x_{j,\pi}$$

$$\sum_{\pi \in \Pi_j} x_{j,\pi} = 1 \qquad \forall j \in J \qquad (3.1)$$

$$v_t - v_{t-1} = 3600\, \Delta t\, (I_t - \sum_{j \in J} \sum_{\pi \in \Pi_j} \sum_{n_t \in N_{jt}} b(n_t) q(n_t) x_{j,\pi}) \qquad \forall t \in T \qquad (3.2)$$

$$\sum_{j \in J} \sum_{\pi \in \Pi_j} \sum_{\substack{n_t \in N_{jt}: \\ b(n)=-1}} x_{j,\pi} \leq \overline{n} - 1 \qquad \forall t \in T \qquad (3.3)$$

$$- \sum_{j \in J} \sum_{\pi \in \Pi_j} \sum_{n_t \in N_{jt}} b(n_t) q(n_t) x_{j,\pi} + \sum_{j \in J} \sum_{\pi \in \Pi_j} \sum_{\substack{n_{t-1} \in \\ N_{j(t-1)}}} b(n_{t-1}) q(n_{t-1}) x_\pi \leq \Delta Q^- \qquad \forall t \in T \qquad (3.4)$$

$$\sum_{j \in J} \sum_{\pi \in \Pi_j} \sum_{n_t \in N_{j(t)}} b(n_t) q(n_t) x_\pi - \sum_{j \in J} \sum_{\pi \in \Pi_j} \sum_{\substack{n_{t-1} \in \\ N_{j(t-1)}}} b(n_{t-1}) q(n_{t-1}) x_{j,\pi} \leq \Delta Q^+ \qquad \forall t \in T \qquad (3.5)$$

$$0 \leq x_{j,\pi} \leq 1, \quad x_{j,\pi} \in \{0,1\} \qquad \forall \pi \in \Pi_J, j \in J \qquad (3.6)$$

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9. Turbines and pumps status

$$g_{jt} + u_{kt} \leq 1 \, \forall j,k \in J, t \in T, \qquad \forall t \in T.$$

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- $|\Pi_j| \neq 0, \forall j \in J$

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Dual variables

$$min \sum_{j \in J} \sum_{\pi \in \Pi_j} c_\pi x_{j,\pi}$$

$$\sum_{\pi \in \Pi_j} x_{j,\pi} = 1 \qquad \forall j \in J \qquad \gamma_j \qquad (3.7)$$

$$v_t - v_{t-1} = 3600\,\Delta t\,(I_t - \sum_{j \in J} \sum_{\pi \in \Pi_j} \sum_{n_t \in N_{jt}} b(n_t) q(n_t) x_{j,\pi}) \qquad \forall t \in T \qquad \alpha_t \qquad (3.8)$$

$$\sum_{j \in J} \sum_{\pi \in \Pi_j} \sum_{\substack{n_t \in N_{jt}: \\ b(n)=-1}} x_{j,\pi} \leq \overline{n} - 1 \qquad \forall t \in T \qquad \beta_t \qquad (3.9)$$

$$-\sum_{j \in J} \sum_{\pi \in \Pi_j} \sum_{n_t \in N_{jt}} b(n_t) q(n_t) x_{j,\pi} + \sum_{j \in J} \sum_{\pi \in \Pi_j} \sum_{\substack{n_{t-1} \in \\ N_{j(t-1)}}} b(n_{t-1}) q(n_{t-1}) x_\pi \leq \Delta Q^- \qquad \forall t \in T \qquad \mu_t \qquad (3.10)$$

$$\sum_{j \in J} \sum_{\pi \in \Pi_j} \sum_{n_t \in N_{j(t)}} b(n_t) q(n_t) x_\pi - \sum_{j \in J} \sum_{\pi \in \Pi_j} \sum_{\substack{n_{t-1} \in \\ N_{j(t-1)}}} b(n_{t-1}) q(n_{t-1}) x_{j,\pi} \leq \Delta Q^+ \qquad \forall t \in T \qquad \delta_t \qquad (3.11)$$

$$0 \leq x_{j,\pi} \leq 1, \quad x_{j,\pi} \in \{0,1\} \qquad \forall \pi \in \Pi_J, j \in J \qquad (3.12)$$

Pricing problem for unit $j \in J$

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and such that

$$" g_{jt} + u_{kt} \leq 1, \; \forall j, k \in J, t \in T"$$

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- Consider an arc $a = (i, k) \in A_j$ characterized by $t, b(i), b(k)$, where $i \in N_{t-1}$, $k \in N_t$,

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- To the weight c_a associated with this arc, we add the following values

$$q(k)(3600\Delta_t \alpha_t - (\delta_t - \mu_t))$$

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$$+\frac{-\mathrm{sign}(q(k))(1-\mathrm{sign}(q(k)))}{2}\beta_t$$

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- Solve a Shortest Path Problem in G_j using Bellman-Ford (as the new weights may be negative),
- Add $-\gamma_j$.

Second step : only a turbine or a pump at once

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$$\overline{c}_\pi = c_\pi - \gamma_j - \sum_t (-3600\Delta_t \alpha_t \sum_{n_t} b(n_t)q(n_t) + \beta_t \sum_{\substack{n_t: \\ b(n)=-1}} 1)$$

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- for each edge $e = (\pi_i, \pi_l) \in E$, we identify the following valid inequality $x_{j,\pi_i} + x_{k,\pi_l} \leq 1$
- this can be generalized for each clique C in H

  $\sum_{\pi \in C} x_\pi \leq 1, \qquad \forall C \subset V$

Conclusion and future work

1. Survey
2. HUC Problem : MILP and path-based formulation
3. Pricing problem

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2. HUC Problem : MILP and path-based formulation
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4. Column generation based heuristic
5. Feasibility recovery heuristic
6. Benchmark for the deterministic HUC problem

Thank you !

Short-term Unit Commitment Problem in Hydro Valleys: Overview and Reformulations

R. TAKTAK [1]

Joint work with:

W. van Ackooij [2], C. D'Ambrosio [1], A. Frangioni [3], C. Gentile [4]

Séminaire PGMO december 16^{th} 2014

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