

Optimal regulations-pricing rules for a wholesale electricity market

Alejandro Jofré¹

Centro de Modelamiento Matemático & Departamento de Ingeniería Matemática
Universidad de Chile

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¹In collaboration with J. Escobar and N. Figueroa 

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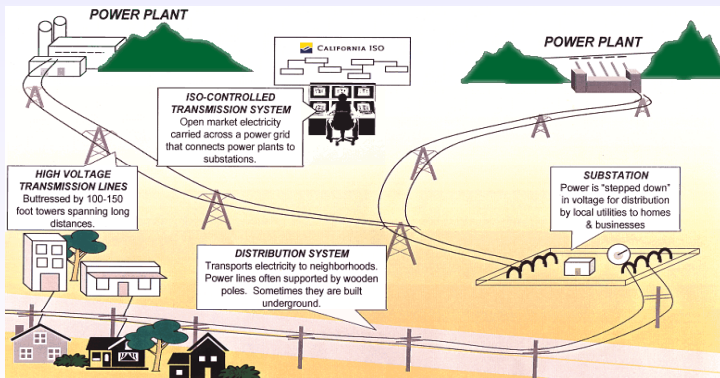
- Introduction and motivation
- Modeling and Market Power
- Efficient regulations and mechanism design
- Conclusions

Since the liberalization of the energy markets in the 80's, they have been modified or improved to avoid market power

How It Works - The California ISO



How It Works



California

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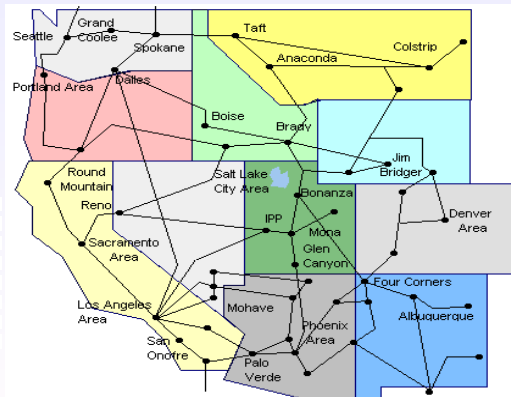
Electricity System Structure



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Electricity System Structure

The System Structure

- Hydropower
- Gas - Steam
- ◆ Combined Cycle
- ◆ Turbine - Gas
- Coal
- ▲ Nuclear
- ◆ Turbine - Oil
- Geothermal



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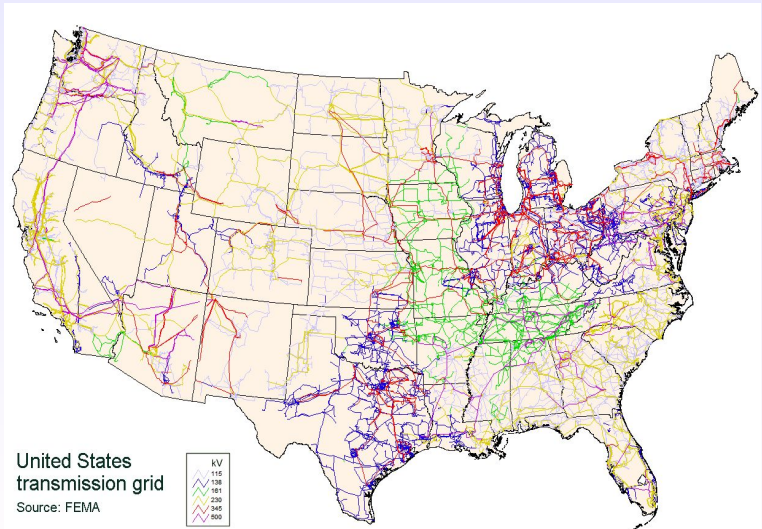
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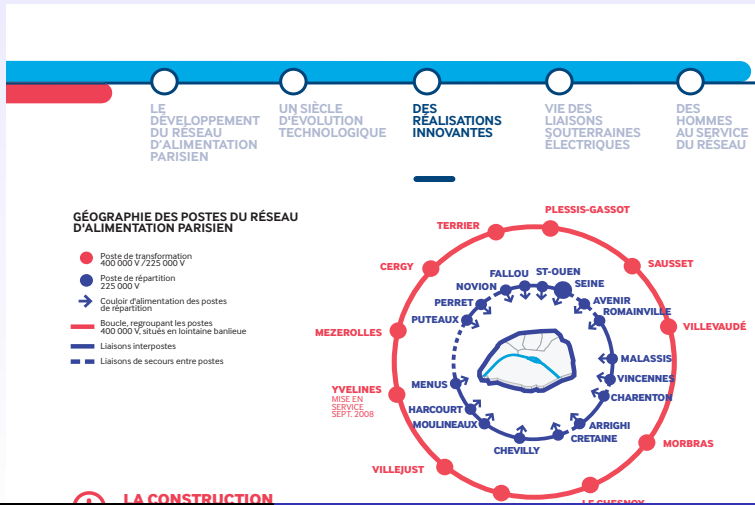
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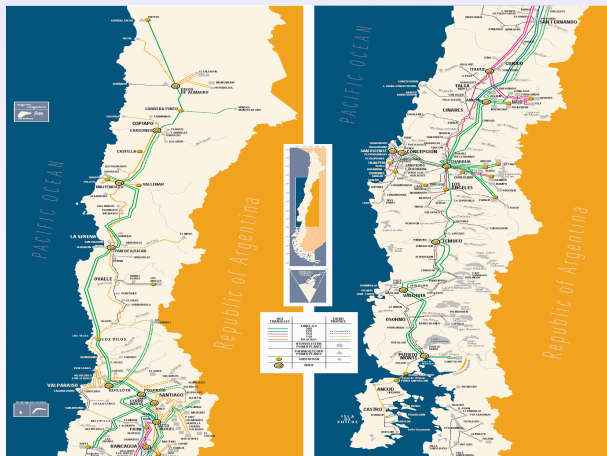
Transmission Chile

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Introduction and Motivation



A generation-transmission pool market

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- Short term: for example day-ahead markets
- Today: generators reveal *generation cost functions* taking into account an estimation of the demand. Generators bid *increasing piece-wise linear cost functions or equivalently piece-wise constant "price"*. Even general convex cost functions.
- Tomorrow: the (ISO) using this information and knowing a realization of the demand, minimizes the sum of the costs to satisfy demands at each node considering all the transmission constraints: "dispatch problem".
- The (ISO) sends back to generators the optimal quantities and "prices" (multipliers associated to supply = demand balance equation at each node)

ISO problem

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The (ISO) knows a realization of the demand $d \in R^V$, receives the costs functions bid $(c_i)_{i \in G}$ and compute how much each generator will produce $(q_i)_{i \in G}$ and the system of "prices-multipliers" $(p_i)_{i \in G}$ solving the following "dispatch" problem:

$$DP(c, d) \quad OPT(c, d) = \min_{(h, q) \in \Omega(d)} \sum_{i \in G} c_i(q_i).$$

In which, for each demand vector d , we encapsulate the supply $\geq$ demand and capacity constraints in:

$$\Omega(d) \subset R^E \times R^G,$$

defined by the following constraints (1)-(2).

modeling ISO: supply-demand constraints

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A dispatch $(h, q) \in \mathbf{R}^E \times \mathbf{R}^G$ is feasible when (Node balance)

$$\sum_{e \in K_i} \frac{r_e}{2} h_e^2 + d_i \leq q_i + \sum_{e \in K_i} h_e \text{sgn}(e, v), \quad v \in G \quad (1)$$

It is called DC approximation.

ISO: Capacity on generation and transmission lines constraints

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$$q_i \in [0, \bar{q}_i], \quad v \in G, \quad (2)$$

where $\bar{q}_i > 0$.

$$0 \leq h_e \leq \bar{h}_e$$

.

We denote $Q(c, d) \subset \mathbf{R}^G$ the generation component of the optimal solution set associated to each cost vector submitted $c = (c_i)$ and demand d .

We denote $\Lambda(c, d) \subset \mathbf{R}^G$ the set of multipliers associated to the node balance (1) in the problem $DP(c, d)$

Modeling Generators

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At each node $v \in G$ we have a generator with payoff

$$u_i(p, q) = pq - \bar{c}_i(q)$$

, in which $\bar{c}_i$ is the real cost. The strategic set for $v \in G$ denoted S_i is the set of functions $c_i: \mathbf{R} \rightarrow \mathbf{R}_+$ convex, nondecreasing with bounded slope- subgradients in $[0, p^*]$ where p^* is a *price cap*.

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An equilibrium is (q, λ, m) such that q is a selection of $Q(\cdot, \cdot)$ and λ is a selection of $\Lambda(\cdot, \cdot)$ and $m = (m_i)_{i \in G}$ is a mixed-strategy equilibrium of the generator game in which each generator submits costs $c_i \in S_i$ with a payoff

$$\mathbb{E} u_i(\lambda_i(c, \cdot), q_i(c, \cdot)) = \int_D u_i(\lambda_i(c, d), q_i(c, d)) d\mathbb{P}(d),$$

where

$$u_i(\lambda_i(c, d), q_i(c, d)) = \lambda_i(c, d) q_i(c, d) - \bar{c}_i(q_i(c, d))$$

Two-node case

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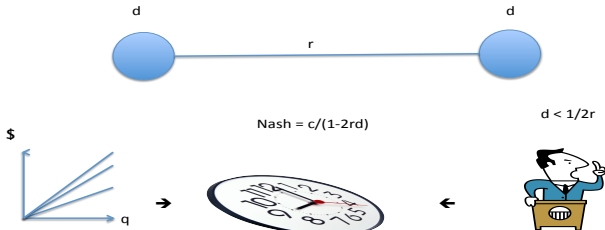
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Two nodes case

Symmetric Nash equilibrium

Profit = multiplier $\times$ quantity - cost $\times$ quantity



Introduction and Motivation: the ISO Problem

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Given that each generator reveals a cost c_i , the (ISO) solves the following minimization problem whose optimal value is denoted $OPT(c, d)$

$$\begin{aligned} \min_{q, h} \quad & \sum_{i=1}^2 c_i q_i \\ \text{s.t.} \quad & q_i - h_i + h_{-i} \geq \frac{r}{2} [h_1^2 + h_2^2] + d \text{ for } i = 1, 2 \\ & q_i, h_i \geq 0 \text{ for } i = 1, 2 \end{aligned}$$

Introduction and Motivation: result

- Escobar and J. (ET (2010) and MOR (2009)), in a symmetric model with complete information, establish that in the presence of transmission costs, equilibrium exists but producers charge a price above marginal cost with the current regulation.
- demand d , r resistance, $c(q) = cq$, $\bar{c}$ is the real cost,

$$Nash = \bar{c} / (1 - 2rd)$$

Economic Intuition

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- This model might also be viewed as an extension of a Bertrand Game
- The losses in the transmission lines induce a product differentiation among generators: Q is continuous.
- **But...** Q_i is non-differentiable. So, Λ is a set valued map and therefore when cannot use the usual tools of oligopoly theory.

Sensitivity formula

Proposition

Let $c \in \prod_{i \in G} S_i$ and $c_i - \hat{c}_i$ a Lipschitz function with constant κ .
Then,

$$|Q_i(c, d) - Q_i(\hat{c}_i, c_{-i}, d)| \leq \kappa \eta,$$

where $\eta = 2 \frac{(1+r_i \bar{h}_i)^2}{\min_{i \in G} r_i c_i^+(0)} \in]0, +\infty[$ and

$$c_i^+(0) = \lim_{y \rightarrow 0+} \frac{c_i(y) - c_i(0)}{y}.$$

Why? Because of the losses, the second-order growth condition is satisfied.

Market Power formula

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Dangerous incentive: If the number of generators is small or the topology of the network isolates some demand nodes then the generators will play strategically with the ISO exercising market power.

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Proposition

The equilibrium prices p_i satisfy

$$\mathbb{E}|p_i - \gamma| \geq \frac{\mathbb{E}[Q_i(p_i, p_{-i}, d)]}{\bar{\eta}}$$

where $\bar{\eta}_i = 2 \frac{|K_i|^2 (1 + \max\{r_e \bar{h}_e : e \in K_i\})^2}{p^ \min_{e \in K} r_e}$ and*

$\gamma(p_{-i}, d)$ is a measurable selection of the subdifferential $\partial \bar{c}_i(Q_i(p_i, p_{-i}, d))$. If for example the true costs are linear, $\bar{c}_i(q) = \bar{c}_i q$, then

$$p_i - \bar{c}_i \geq \frac{\mathbb{E}[Q_i(p_i, p_{-i}, d)]}{\bar{\eta}}.$$

The Questions

In an electric network with transmission costs and private information:

- Does the usual (price equal Lagrange multiplier) regulation mechanism minimize costs for the society?
- If not, what is the mechanism that achieves this objective?
- How does the performance of both systems compare?

Methodology:

- Bayesian Game Theory
- Mechanism Design

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- Two-node network with demand d at each node.
- One producer at each node, with marginal cost of production $c_i \sim F_i[\underline{c}_i, \bar{c}_i]$.
- Transmission costs rh^2 , with h the amount sent from one node to another.

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The ISO Problem

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Given that each generator reveals a cost c_i , the dispatcher solves:

$$\begin{aligned} \min_{q,h} \quad & \sum_{i=1}^2 c_i q_i \\ \text{s.t.} \quad & q_i - h_i + h_{-i} \geq \frac{r}{2} [h_1^2 + h_2^2] + d \text{ for } i = 1, 2 \\ & q_i, h_i \geq 0 \text{ for } i = 1, 2 \end{aligned}$$

The Solution for ISO problem

If we define

$$H(x, y) = d + \frac{1}{2r} \left(\frac{x - y}{x + y} \right)^2 - \frac{1}{r} \left(\frac{x - y}{x + y} \right)$$

and

$$\bar{q} = 2 \left[\frac{1 - \sqrt{1 - 2dr}}{r} \right]$$

then the solution to this problem can be written as

$$q_i(c_i, c_{-i}) = \begin{cases} H(c_i, c_{-i}) & \text{if } H(c_i, c_{-i}) \geq 0 \text{ and } H(c_{-i}, c_i) \geq 0 \\ \bar{q} & \text{if } H(c_{-i}, c_i) < 0 \\ 0 & \text{if } H(c_i, c_{-i}) < 0 \end{cases}$$
$$\lambda_i(c_i, c_{-i}) \equiv p_i(c_i, c_{-i}) = c_i \text{ if } H(c_i, c_{-i}) \geq 0$$

The Bayesian Game

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The game:

- 2 players. Strategies $c_i \in C_i = [\underline{c}_i, \bar{c}_i]$, $i=1,2$.
- Payoff $u_i(c_i, c_{-i}) = (p_i(c_i, c_{-i}) - \mathbf{c}_i)q_i(c_i, c_{-i})$,

where $\mathbf{c}_i$ is the real cost. The Equilibrium:

- A strategy $b : [\underline{c}_i, \bar{c}_i] \longrightarrow [\underline{c}_i, \bar{c}_i]$.
- In a Nash equilibrium

$$\bar{b}(c) \in \arg \max_x \int_{C_{-i}} [p_i(x, \bar{b}(c_{-i})) - c] q_i(x, \bar{b}(c_{-i})) f_{-i}(c_{-i}) dc_{-i} \quad (3)$$

Numerical Approximation

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- For simplicity $C_i = [1, 2]$.
- Let $k \in \{0, \dots, n-1\}$, and $b(c) = b_k$ for $c \in [\frac{k}{n}, \frac{k+1}{n}]$.
- The weight of each interval is given by $w_k = F(\frac{k+1}{n}) - F(\frac{k}{n})$.
- The approximate equilibrium is characterized by:

$$b_k \in \arg \max_x \sum_{l=0}^{n-1} [p_i(x, b_l) - r_k] q_i(x, b_l) w_l \text{ for all } k \in \{0, \dots, n-1\} \quad (4)$$

Optimal Mechanism

Mechanisms

- A *direct revelation mechanism* $M = (q, h, x)$ consists of an *assignment rule* $(q_1, q_2, h_1, h_2) : C \rightarrow R^4$ and a *payment rule* $x : C \rightarrow R^2$.
- The ex-ante expected utility of a buyer of type c_i when he participates and declares c'_i is

$$U_i(c_i, c'_i; (q, h, x)) = E_{c_{-i}}[x_i(c'_i, c_{-i}) - c_i q_i(c'_i, c_{-i})]$$

- A mechanism (q, h, x) is feasible iff:

$$U_i(c_i, c_i; (q, h, x)) \geq U_i(c_i, c'_i; (q, h, x)) \text{ for all } c_i, c'_i \in C_i$$

$$U_i(c_i, c_i; (q, h, x)) \geq 0 \text{ for all } c_i \in C_i$$

$$q_i(c) - h_i(c) + h_{-i}(c) \geq \frac{r}{2}[h_1^2(c) + h_2^2(c)] + d \text{ for all } c \in C$$

$$q_i(c), h_i(c) \geq 0 \text{ for all } c \in C$$

The Regulator's Problem

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Using the revelation principle, the regulator's problem can be written as:

$$\min_c \int \sum_{i=1}^2 x_i(c) f(c) dc \quad (5)$$

subject to (q, h, x) being “feasible”

The Regulator's Problem (II)

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It can be rewritten as

$$\begin{aligned} \min \quad & \int_C \sum_{i=1}^2 q_i(c) \left[c_i + \frac{F_i(c_i)}{f_i(c_i)} \right] f(c) dc \\ \text{s.t} \quad & \int_{C_{-i}} q_i(c_i, c_{-i}) f_{-i}(c_{-i}) dc_{-i} \text{ is non-increasing in } c_i \\ & q_i(c) - h_i(c) + h_{-i}(c) \geq \frac{r}{2} [h_1^2(c) + h_2^2(c)] + d \text{ for all } c \in C \\ & q_i(c), h_i(c) \geq 0 \text{ for all } c \in C \end{aligned}$$

We denote by $J_i(c_i) = c_i + \frac{F_i(c_i)}{f_i(c_i)}$ the virtual cost of agent i . We assume it is increasing (Monotone likelihood ratio property: true for any log concave distribution)

Solution

An optimal mechanism is given by

$$\hat{q}_i(c_i, c_{-i}) = \begin{cases} H(J_i(c_i), J_{-i}(c_{-i})) & \text{if } H(J_i(c_i), J_{-i}(c_{-i})) \geq 0 \text{ and} \\ \bar{q} & \text{if } H(J_{-i}(c_{-i}), J_i(c_i)) < 0 \\ 0 & \text{if } H(J_i(c_i), J_{-i}(c_{-i})) < 0 \end{cases}$$

$$\hat{x}_i(c_i, c_{-i}) = c_i \hat{q}_i(c_i, c_{-i}) + \int_{c_i}^{\bar{c}_i} \hat{q}_i(s, c_{-i}) ds$$

Such a mechanism is dominant strategy incentive compatible.

Comparison

We consider the family of distributions with densities

$$f_a(x) = \begin{cases} a(x-1) + (1 - \frac{a}{4}) & \text{if } x \leq 1.5 \\ -a(x-1) + (1 + \frac{3a}{4}) & \text{if } x \geq 1.5 \end{cases}$$

Asymmetric information

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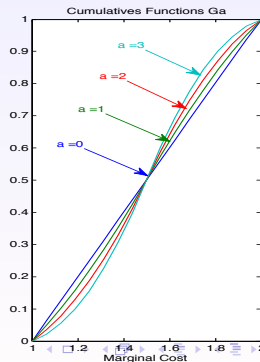
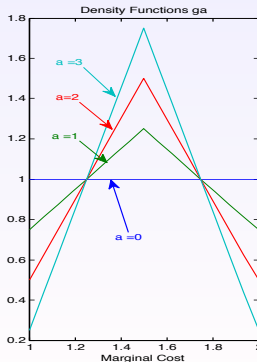
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Social costs for different mechanisms

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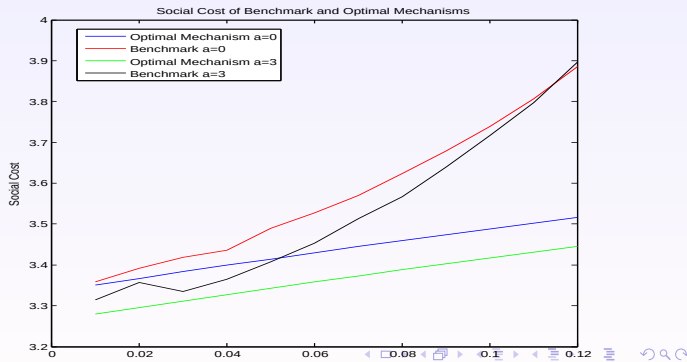
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Robustness and Practical Implementation

- The optimal mechanism is detail free. If the designer is wrong about common beliefs, then the mechanism is still not bad:

$$\|X_f - X_{\tilde{f}}\| \leq \|x\|_1 \|f - \tilde{f}\|_\infty \leq \bar{c}\bar{q} \|f - \tilde{f}\|_\infty$$

- The assignment rule is computationally simple to implement. It requires solving **once** the dispatcher problem, with modified costs.
- However, the payments are computationally difficult

$$c_i \hat{q}_i(c_i, c_{-i}) + \int_{c_i}^{\bar{c}_i} \hat{q}_i(s, c_{-i}) ds$$

- The integral requires solving infinitely many dispatcher problems. But it can be approximated using the risk neutrality of agents.



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