Unité de Mathématiques Appliquées de l'ENSTA

Subject: Non-destructive testing in elasto-plasticity. An optimal control approach.

1 Description of the subject

Let us assume that a metal structure which is initially load free undergoes external loading that creates plastic deformations inside it. An observer who knows nothing about the history of this loading subsequently observes a residual deformation of the structure when it is once again load free. This observer then wishes to determine the history of the load based on this single observation, or at least the plastic deformations inside the structure created by this load history. This is a non-destructive testing problem, the solution to which would determine whether the structure is healthy enough to remain in service or should be replaced. Several applications come to mind. One example is an underground pipe that undergoes a landslide, causing it to deform plastically. Another is an airplane wing that has been struck by a tool, causing a visible dent.

A natural idea for solving this problem is to formulate it as an optimal control problem with final surface observation and surface control over the entire time interval. The main difficulty then lies in the fact that the elasto-plastic flow law is governed by a differential inclusion and not a differential equation. In the very recent article [4], the case of a beam in pure bending was studied and solved, the simplicity of the geometry in this case allowing the direct problem to be partially solved "by hand" in the thickness of the beam, which made it possible to circumvent the problem of differential inclusion and brought us back to a one-dimensional optimal control problem (along the axis of the beam). The task now is to deal with more realistic and therefore more complicated geometries. To this end, we plan to regularize the elasto-plastic behavior law into a visco-plastic law, using a regularization parameter that transforms the differential inclusion into a differential equation. If the visco-plastic potential is sufficiently regular, the equations of state are differentiable and the gradient of the cost function can be calculated using an adjoint state, as is traditionally done in optimal control. In the case of the elasto-plastic law under consideration, hardening will be taken into account, not only because metals are indeed work hardenable, but also because this hardening ensures that the direct problem is well-posed. The simplest type of hardening will be used initially, namely linear kinematic hardening. In addition, problems of increasing difficulty will be considered. We can start with the simple problem studied in [4], no longer solving the direct problem "by hand" in thickness to reduce it to a 1D problem, but directly in 2D using viscoplastic regularization. We can then limit ourselves to antiplane elasto-plasticity in order to reduce the number of unknowns, and finally look at a real 2D or 3D elasto-plasticity problem.

The subject is innovative in that the problem of non-destructive testing itself (not just the method of resolution) has never been addressed in the past. The idea of reconstructing the loading path that led to residual displacements due to plasticity is proposed in [1], but has never been seriously implemented numerically since, except for a simple case in the article [4] already cited. It is also worth mentioning the articles [2, 3] on the shape optimization of elasto-plastic structures, which has points in common with our problem. In our opinion, the main reason for this lack of interest is not a lack of potential applications, but rather the intrinsic difficulty of the problem linked to the differential inclusion governing the time derivative of plastic deformation. In [4], we found that the reconstruction of the loading path suffers from a very strong non-uniqueness, which makes a precise reconstruction of the loading path impossible. On the other hand, uniqueness does occur for plastic strains, which are the quantities of interest for Non-Destructive Testing.

2 Prerequisites

This topic requires dual expertise in applied mathematics (analysis of PDEs, functional analysis, numerical analysis, scientific programming) and structural mechanics.

3 Supervision

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References

[1] H. D. Bui, *Introduction aux problèmes inverses en mécanique des matériaux*, Eyrolles, 1993.

[2] A. Maury, G. Allaire and F. Jouve, Elasto-plastic shape optimization using the level set method, *SIAM Journal on Control and Optimization*, **56** (2018), 556–581.

[3] J. Desai, G. Allaire, F. Jouve and C. Mang, Topology optimization in quasi-static plasticity with hardening using a level-set method, *Struct. Multidiscip. Optim.*, **64** (2021), 3163–3191.

[4] L. Bourgeois, J.-F. Mercier, An inverse problem related to an elasto-plastic beam, *Inverse Problems*, to appear.