On Bounded Storage Key Agreement and One-Way Functions

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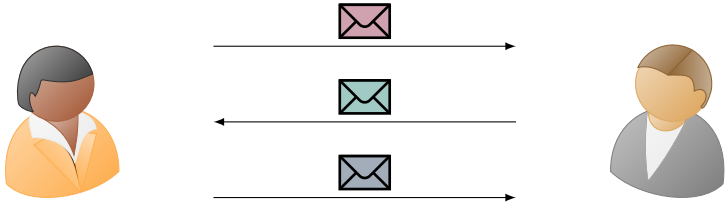
FONDAMENTALE

Université
Paris Cité

Marie Sklodowska-Curie
Math in Greater Paris

Key Agreement



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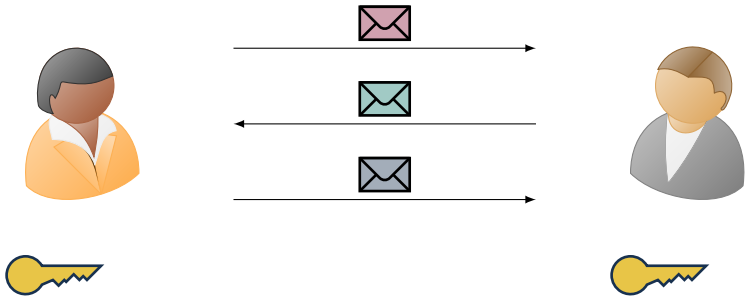

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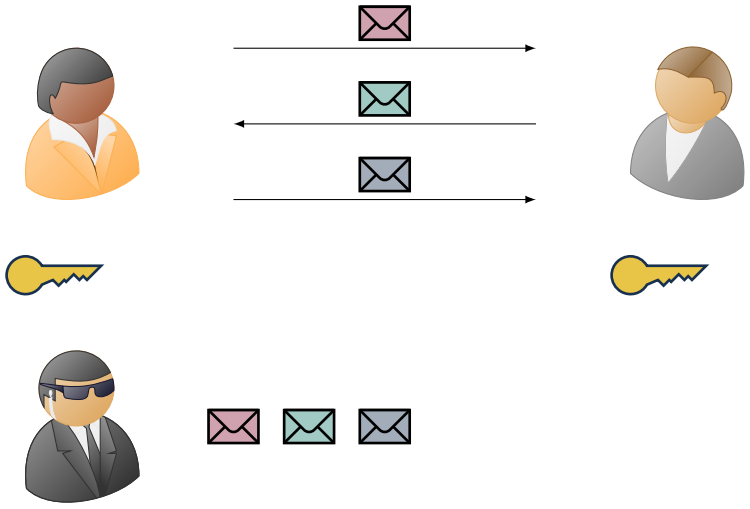

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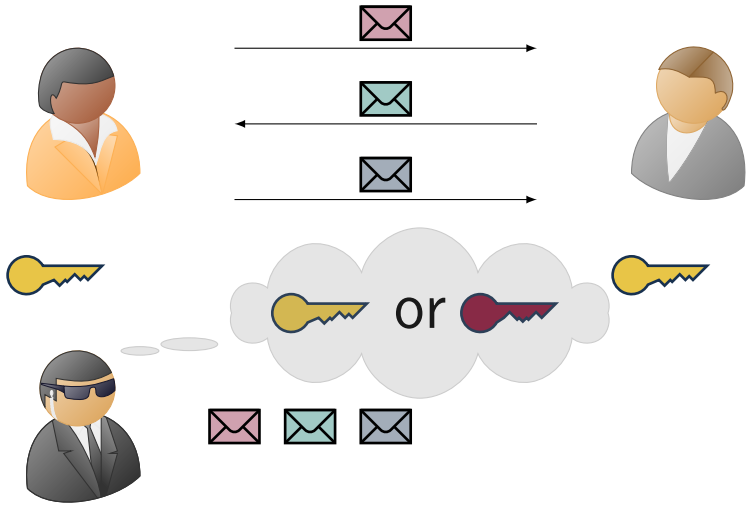

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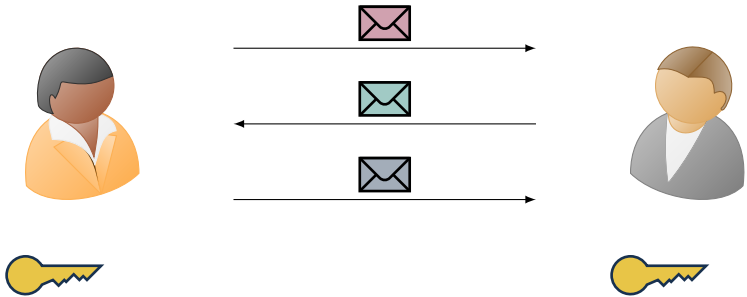

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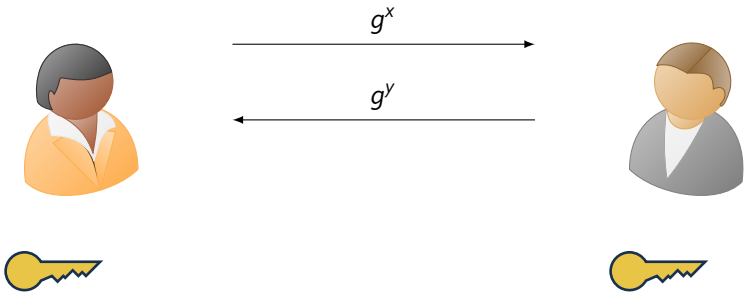
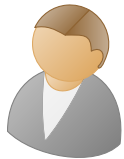


Key Agreement

g^x

g^y



$(g^x)^y = g^{xy}$

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Birthday Bound Protocols

- Key Agreement without Public Key Cryptography
- “Cryptography if there is no Cryptography”

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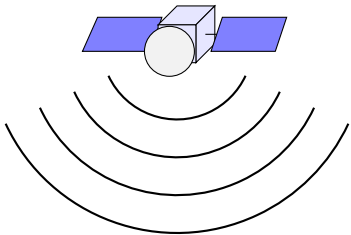
- Here: Restrict Adversary's Memory **and** Runtime

Outline

- Key Agreement in Bounded Space
- …using Computational Assumptions
- …implying Computational Hardness

In Bounded Space [Cachin-Maurer]



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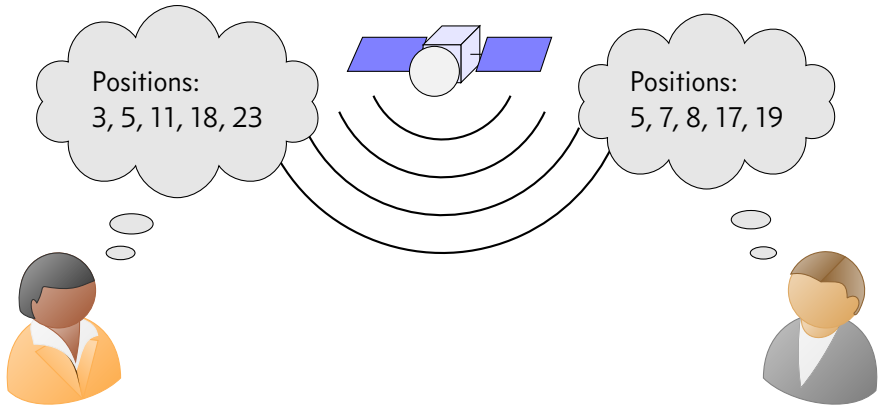

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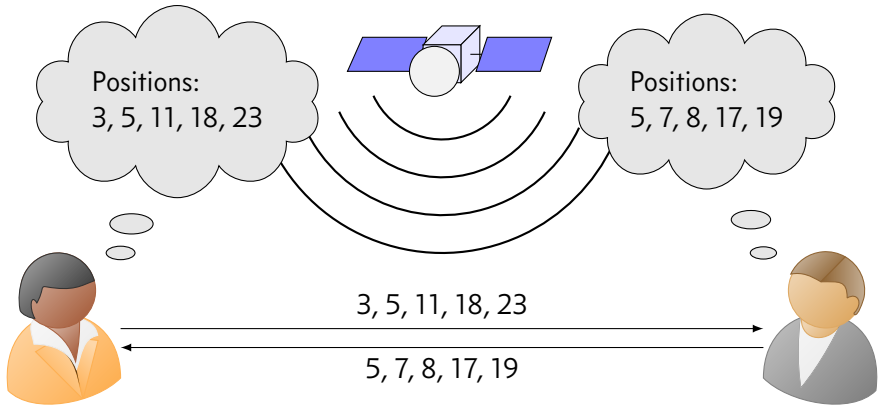

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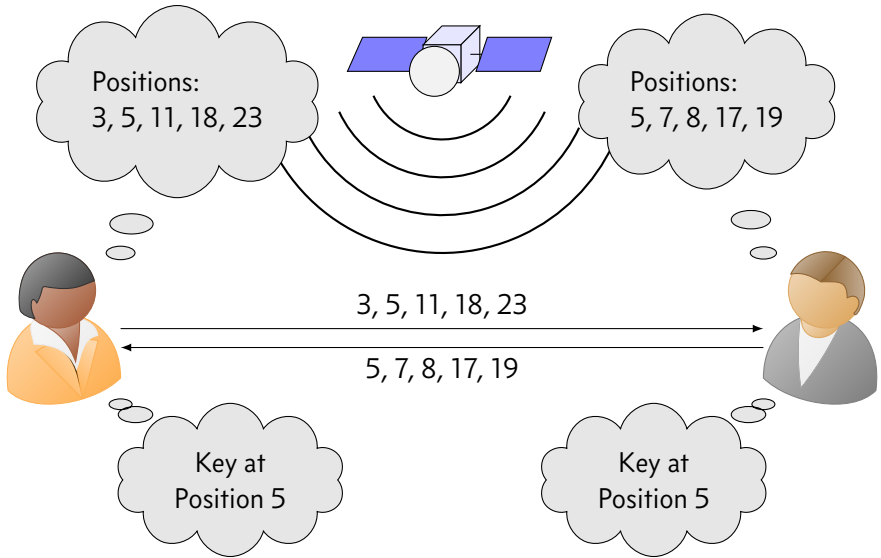
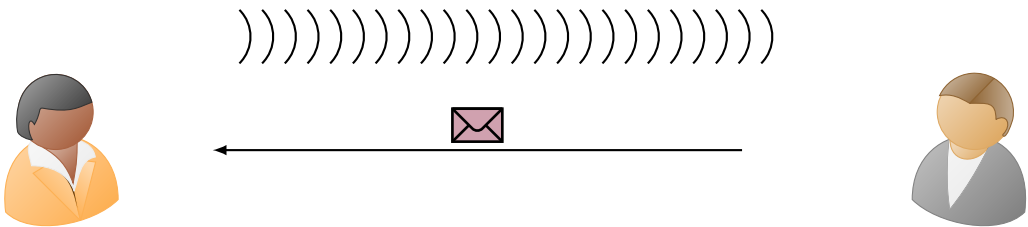


Lower Bounds [Dziembowski-Maurer]



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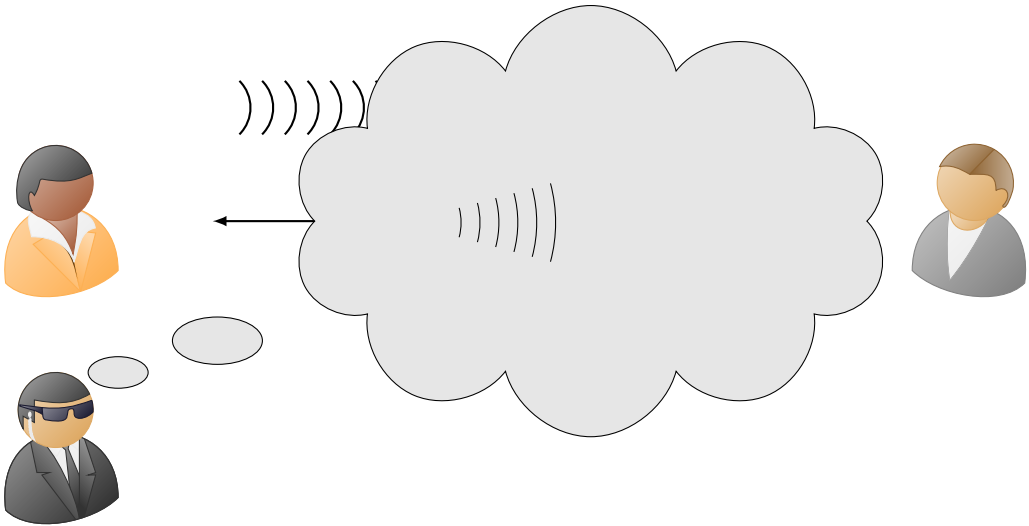

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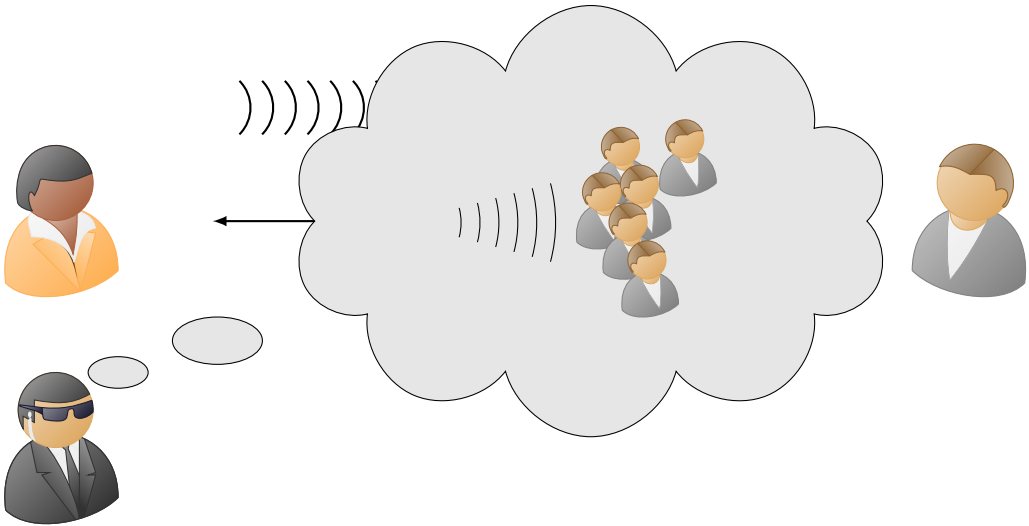


Lower Bounds [Dziembowski-Maurer]

$I($  ,  $)$

becomes large

Information Theoretic Setting

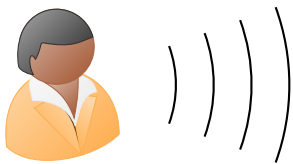
Theorem (Cachin-Maurer)

There exist key agreement protocols in bounded space for honest parties using space s that is secure against adversaries with $o(s^2)$ space.

Theorem (Dziembowski-Maurer)

Any key agreement protocol for parties with space s can be broken in $O(s^2)$ space.

Using Computational Assumptions



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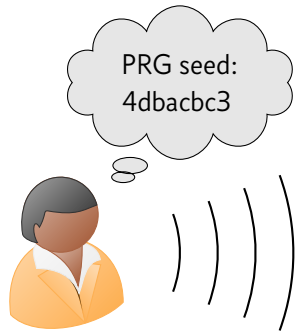

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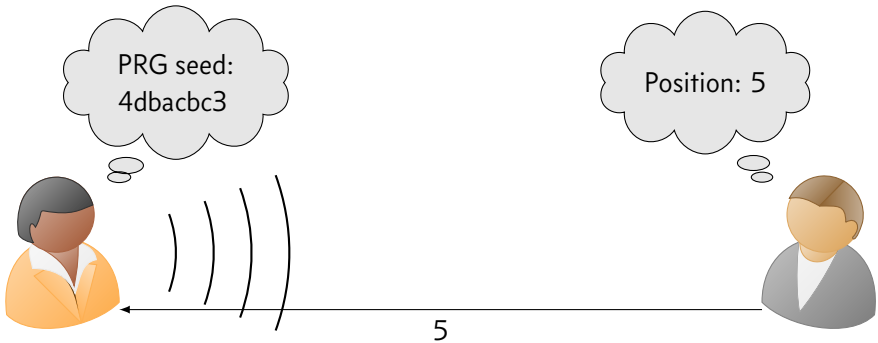

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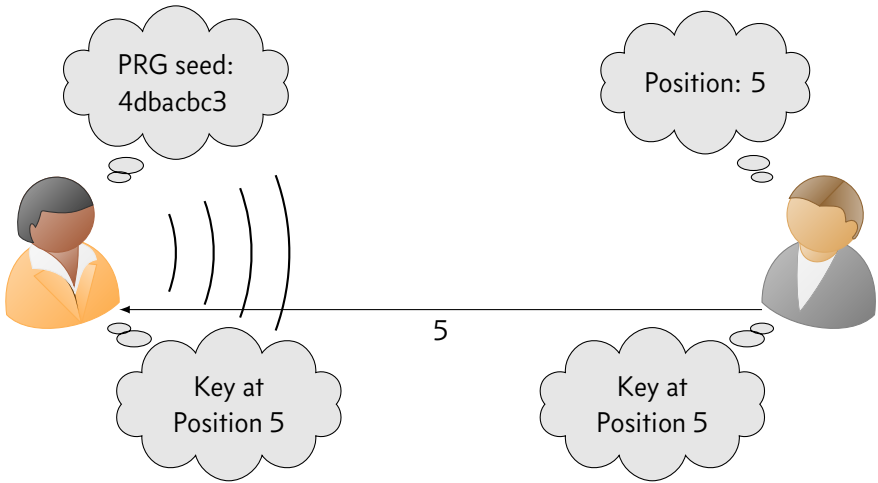

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Theorem (informal)

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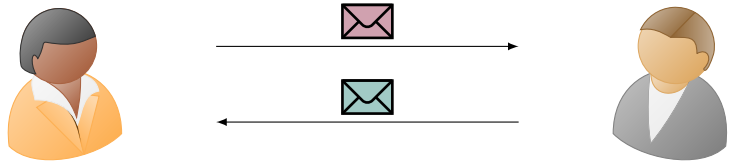
$$\mathrm{KA} \Rightarrow \mathrm{dOWF} \Rightarrow \mathrm{wOWF} \Rightarrow \mathrm{OWF} \Rightarrow \mathrm{PRG} \Rightarrow \mathrm{PRF}$$

(Distributional) One-Way Functions



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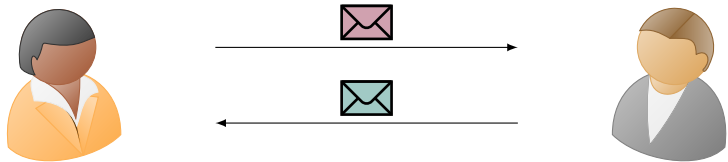

$$f(b, r_A, r_B, k^*) \rightarrow \begin{cases} \text{transcript}, k & \text{if } b = 0 \\ \text{transcript}, k^* & \text{if } b = 1 \end{cases}$$

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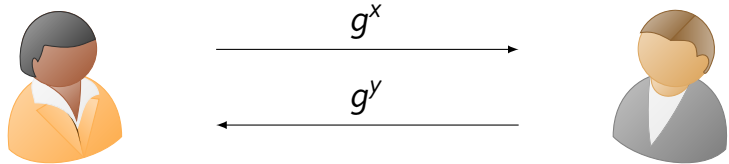

$$f(b, x, y, g^z) \rightarrow \begin{cases} g^x, g^y, g^{xy} \text{ if } b = 0 \\ g^x, g^y, g^z \text{ if } b = 1 \end{cases}$$

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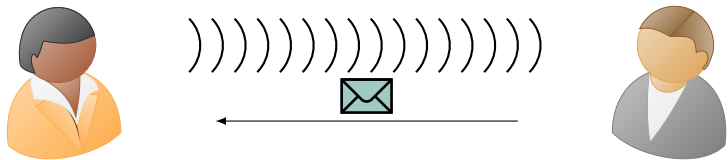

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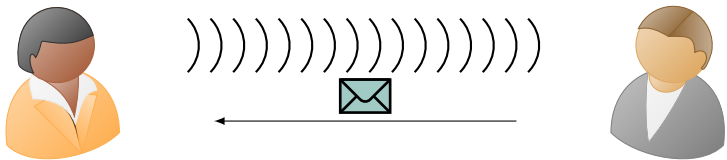

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Summary

Theorem

For every polynomial p, there exist key agreement protocols secure against adversaries running in space $o(p(s))$ and time $\text{poly}(\lambda)$ if and only if One-Way functions exist.