Numerical Shape Optimization among 3D convex sets

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Shape optimization problems have the following generic form:

$$\min_{\Omega \in \mathcal{A}} J(\Omega),$$

i.e. they are optimization problems where the variable is a shape. The objective function $\Omega \mapsto J(\Omega)$ can be related to geometric quantities (perimeter, area, etc) or can depend on Ω via a partial differential equation. The class $\mathcal{A}$ of admissible shapes may include various constraints like the volume, diameter, convexity.

The theoretical study of shape optimization problem under convexity, diameter, constant width, minimal width constraints poses various difficulties on the theoretical side: not all arbitrarily small perturbations preserve these constraints. Therefore, numerical tools were developed in [1], [2] to investigate further such problem and provide leads for the theoretical study. The paper [1] included both 2D and 3D numerical frameworks, but the convexity of the discrete shapes is not validated theoretically. In [2] a completely rigorous numerical framework is proposed in dimension two.

The goal of this project is to extend the results in [2] to the three dimensional case. The objectives of the project are the following:

- Devise a discrete framework which is the three dimensional analogue of [2]. The support function will be parametrized using a finite number of directions on the unit sphere. The convexity constraint will be modeled by a set of linear inequalities for the parameters corresponding to the set of directions.
- Given a set of parameters verifying the previous inequalities, construct a discrete polyhedron which approximates the desired convex shape.
- Prove that the discrete framework is rigorous: it always produces convex shapes. Ideally, the discrete framework should allow to approximate arbitrarily well any 3D convex shape.
- Implement numerically the discretization methods and study various convex optimization problems in 3D.

None of the previously inderlined items is a direct extension of the 2D case, although it is expected that similar ideas should work, with new potential difficulties regarding three dimensional aspects.

The applicant is expected to have previous experience in calculus of variations or shape optimization for both theoretical and numerical aspects. Familiarity with aspects in three dimensional geometry is also necessary. The code can be in one of the following programming languages: Matlab, Python, FreeFEM.

References

[1] P. R. S. Antunes and B. Bogosel. Parametric shape optimization using the support function. *Comput. Optim. Appl.*, 82(1):107–138, 2022.

[2] B. Bogosel. Numerical shape optimization among convex sets. *Applied Mathematics and Optimization*, 87(1), Nov. 2022.