FMJH postdoc proposal

"Tensor RG approach to renormalization of lattice models"

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Abstract

Tensor renormalization group has recently emerged as a new rigorous technique, which uses ideas of quantum information to manipulate and coarse-grain tensor networks representing partition functions of lattice spin models of statistical lattice models, and to detect their phases. This project will be devoted to exploring this idea further.

Proposal

Tensor approach to lattice RG has been developed in theoretical physics literature, as a set of numerical techniques, since 2007 [1]. In this approach, one rewrites a partition function of a spin lattice model as a periodic contraction of tensors—a tensor network. RG transformations consists in performing coarse-graining of the network by rewriting it equivalently in terms of a smaller number of tensors having the same overall contraction value.

In recent years, rigorous approach to tensor RG was pioneered by myself and Tom Kennedy, in two dimensions [2][3]. This approach operates on infinite-dimensional tensors and does not involve any truncation. More recently, we developed it further in [4] ingerating it into a rigorous computer assisted framework. Also, my PhD student Nikolay Ebel proved a theorem about the high-T phase of 3D models using tensor network RG [5]. Hopefully, we will soon prove the existence of a critical fixed point of the 2D Ising model using this approach.

The goal of this project will be to push this field further, towards applications for other lattice models. Ideal candidate for this postdoctoral position will combine ability to do rigorous analysis in infinite dimensions with the willingness to do computer experimentations and computer-assisted proofs. The final results will be rigorous mathematical physics, but computers may have to be used to get there. The expectation is that this method can go much further than other existing techniques, and in particular eventually lead to rigorous results for critical exponents of 3D models, such as the 3D Ising model. There is also space for people who don't want to come near computers, but their work will necessarily be confined to various limiting cases.

Bibliography

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