SOLVING LARGE *CUMULATIVE* SCHEDULING PROBLEMS

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MOTIVATIONS

- **Need to handle large scale problems.**

  [Panel of the Future of CP 2011]
- **(Multi-dimensional) bin-packing problems, in the context of cloud computing.**

  [Panel of the Future of CP 2011], [2012 Roadef Challenge]
- **Existing papers usually leave open the scalability issue.**
- **Time-Table constraint is a good candidate.**

  [Baptiste 2006, Samos] time-tabling used in ILOG Scheduler for scalability purpose

  [Vilim, 2011 CPAIOR]

OUTLINE

The *cumulative* Constraint

A Critical Analysis of the [CP2001] Sweep Algorithm

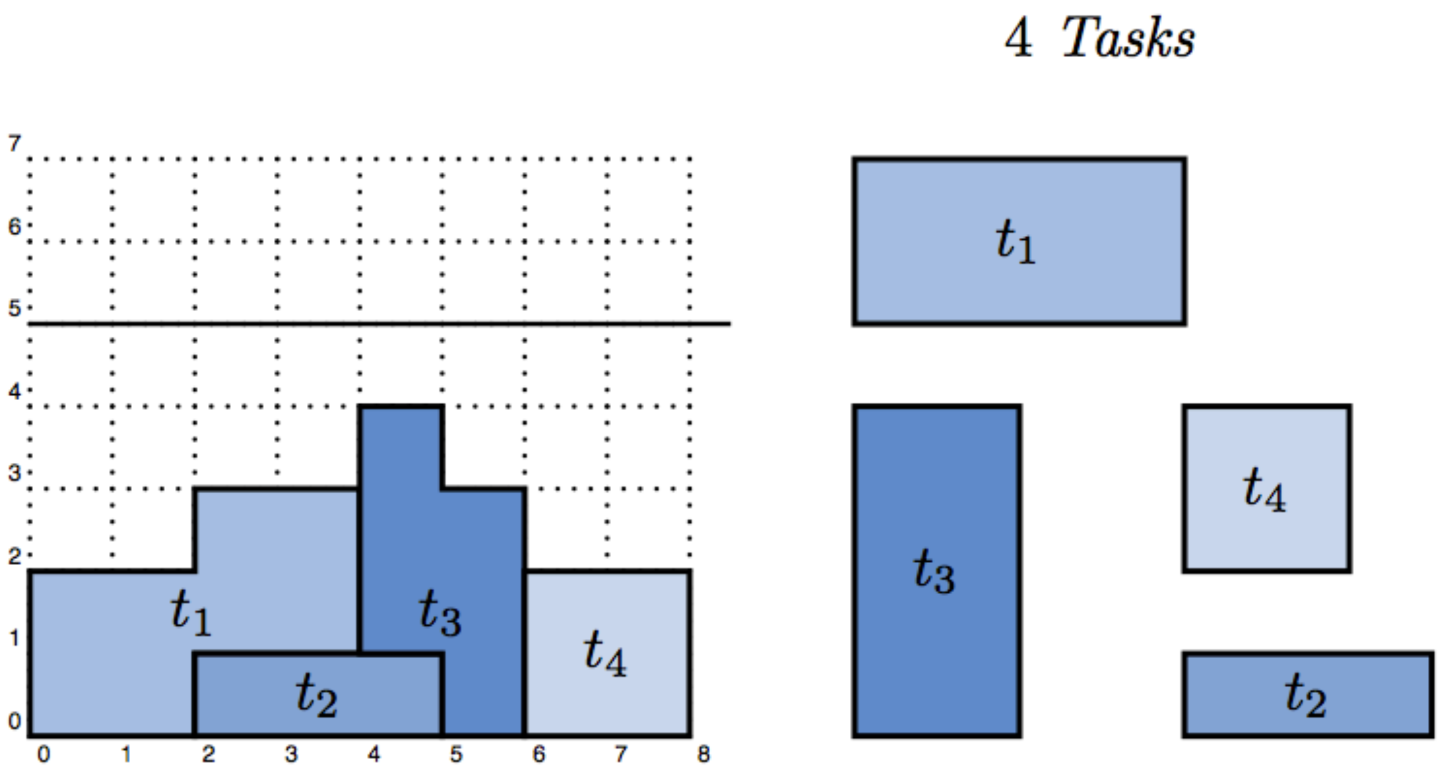
The Dynamic Sweep Algorithm (one *cumulative*)

The Dynamic Sweep Algorithm (several *cumulative* + *precedence*)

Evaluation

Conclusion

THE *CUMULATIVE* CONSTRAINT



OUTLINE

The *cumulative* Constraint

A Critical Analysis of the [CP2001] Sweep Algorithm

- Principle
- Illustration
- 4 Weaknesses

The Dynamic Sweep Algorithm (one *cumulative*)

The Dynamic Sweep Algorithm (several *cumulative* + *precedence*)

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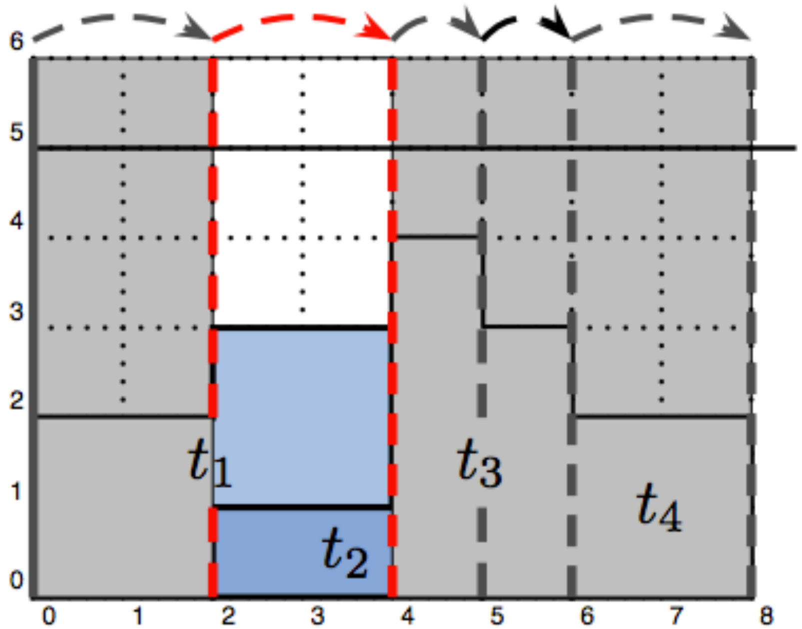
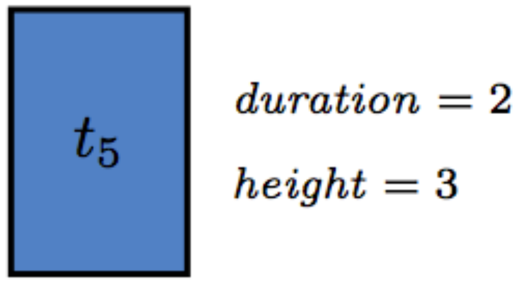
A CRITICAL ANALYSIS OF THE [CP2001] SWEEP ALGORITHM
(OVERVIEW)

The sweep-line "jumps" from event to event in order to build the cumulated profile and to perform checks and pruning.



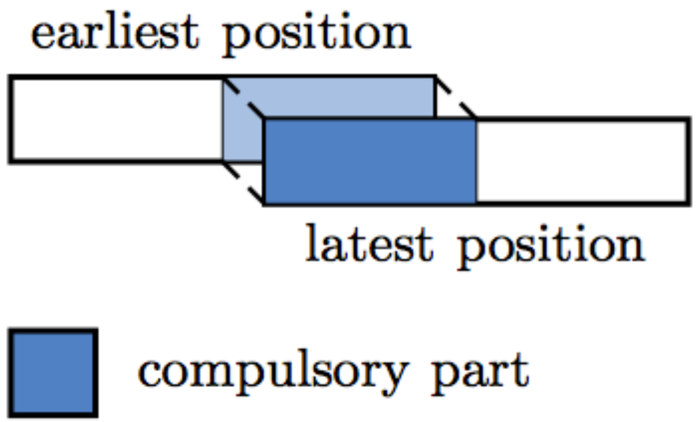


This task cannot overlap [2,4)

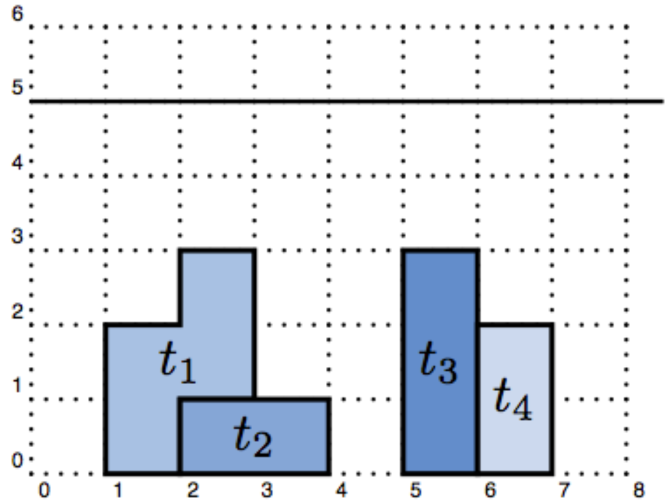
This task cannot start on [1,4)

A CRITICAL ANALYSIS OF THE [CP2001] SWEEP ALGORITHM
(PRINCIPLE: COMPULSORY PARTS)

Compulsory Part : the intersection of all the feasible instances of a task.



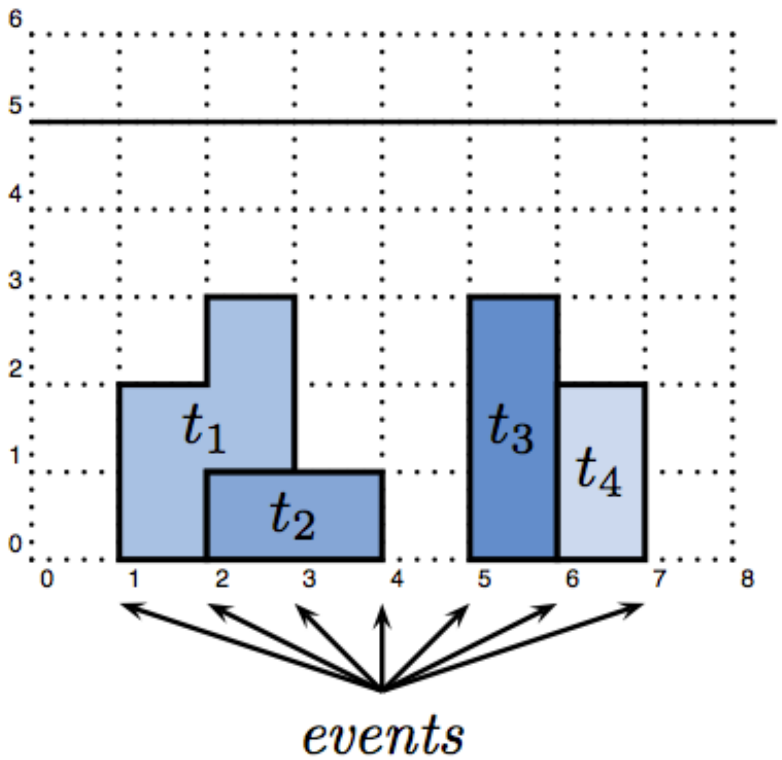
Cumulated Profile: the union of all the compulsory parts.



A CRITICAL ANALYSIS OF THE [CP2001] SWEEP ALGORITHM

(PRINCIPLE: EVENTS)

Event : a potential change of the height of the cumulated profile.
(i.e. start and end of compulsory part)



A CRITICAL ANALYSIS OF THE [CP2001] SWEEP ALGORITHM

(PRINCIPLE)

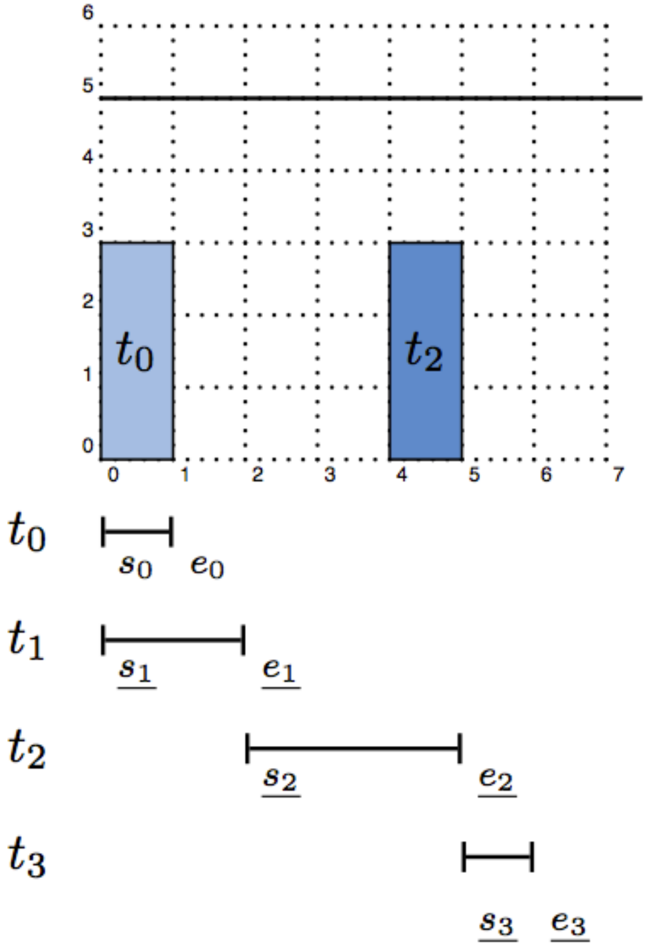
1. All events on the current sweep-line position are read (amount of available resource is updated).

2. The current and the next sweep-line positions define a sweep interval.

3. Scans all tasks that overlap the sweep interval. If the height of a task is greater than the available resource, an interval is removed from the start of the task.

A CRITICAL ANALYSIS OF THE [CP2001] SWEEP ALGORITHM

(ILLUSTRATION: INITIAL PROBLEM)



$$t_0 : s_0 = 0,\ d_0 = 1,\ e_0 = 1,\ h_0 = 3$$

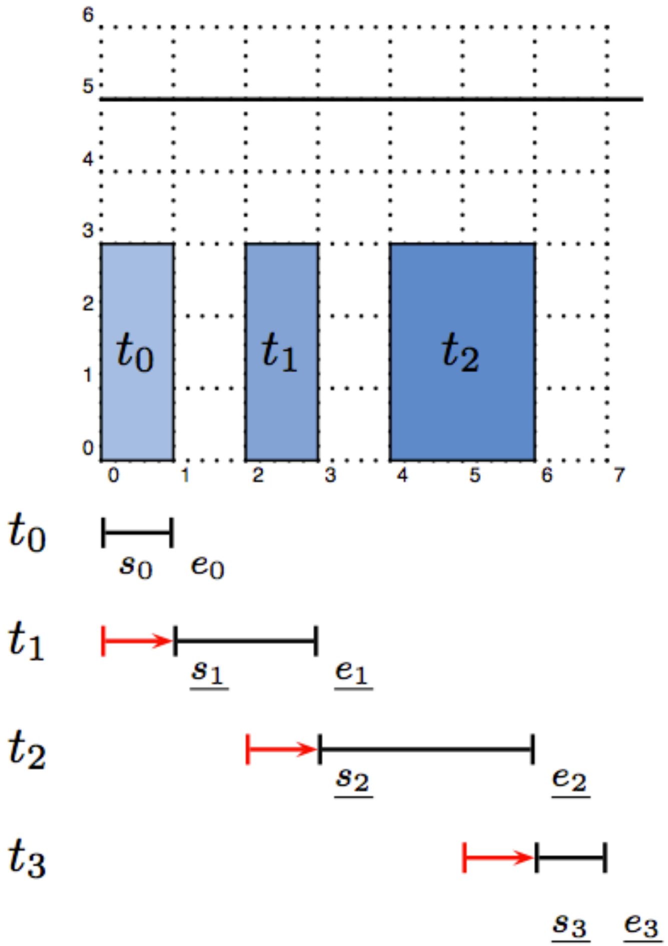
$$t_1 : s_1 \in [0, 2],\ d_1 = 2,\ e_1 \in [2, 4],\ h_1 = 3$$

$$t_2 : s_2 \in [2, 4],\ d_2 = 3,\ e_2 \in [5, 7],\ h_2 = 3$$

$$t_3 : s_3 \in [5, 7],\ d_3 = 1,\ e_3 \in [6, 8],\ h_3 = 3$$

A CRITICAL ANALYSIS OF THE [CP2001] SWEEP ALGORITHM

(ILLUSTRATION: AFTER A FOURTH SWEEP)



After 4 sweeps

$t_0 : s_0 = 0,\ d_0 = 1,\ e_0 = 1,\ h_0 = 3$

$t_1 : s_1 \in [1, 2],\ d_1 = 2,\ e_1 \in [3, 4],\ h_1 = 3$

$t_2 : s_2 \in [3, 4],\ d_2 = 3,\ e_2 \in [6, 7],\ h_2 = 3$

$t_3 : s_3 \in [6, 7],\ d_3 = 1,\ e_3 \in [7, 8],\ h_3 = 3$

A CRITICAL ANALYSIS OF THE [CP2001] SWEEP ALGORITHM
(4 WEAKNESSES)

1. **Too static**:

   Does not take into account the potential increase of the cumulated profile during a single sweep (see previous example).

2. **Often reaches its worst time complexity**:

   It needs to systematically re-scan all tasks that overlap the current sweep-line position to perform pruning. ($O(n^2)$)

3. **Creates holes in the domains**:

   A variable cannot just be compactly represented by its min/max values.

4. **Does not take advantage of the bin-packing**:

   The worst-case time complexity is left unchanged and is often reached.

OUTLINE

The *cumulative* Constraint

A Critical Analysis of the [CP2001] Sweep Algorithm

The Dynamic Sweep Algorithm (one *cumulative*)

- Principle
- Illustration
- Property and Complexity
- Greedy Mode

The Dynamic Sweep Algorithm (several *cumulative* + *precedence*)

Evaluation

Conclusion

THE DYNAMIC SWEEP ALGORITHM
(PRINCIPLE)

1. **A Dynamic sweep based algorithm**:

   It can directly take into account the increase of the cumulated profile during a single sweep.

2. **A “good” average time complexity**:

   Essential in order to handle large instances.

3. **Does not create holes in domains**:

   A variable can be compactly represented by its min/max values.

4. **Takes advantage of the bin-packing**:

   A better worst-case time complexity than for the cumulative.

THE DYNAMIC SWEEP ALGORITHM
(PRINCIPLE)

- Deal with **domain bounds**. [CP2012]
  (**Creates holes in the domains**. [CP2001])
- Filter min and max values in **two distinct sweep stages**: *sweep_min* and *sweep_max*, speeds up the convergence to the fixpoint. [CP2012]
- New **dynamic** and **conditional events** [CP2012]
  (**Too static** [CP2001])
- Use dedicated **data structures**. [CP2012]
  (**Often reaches its worst time complexity** [CP2001])

THE DYNAMIC SWEEP ALGORITHM
(PRINCIPLE: NEW EVENTS)

- **Event related to the end of the compulsory part of a task is now dynamic.**

- **A conditional event is generated for each task initially without compulsory part.**

The adjustment of the earliest start of the task can induce the creation of a compulsory part.

⬇

The conditional event is transformed into 2 events reflecting the new compulsory part.

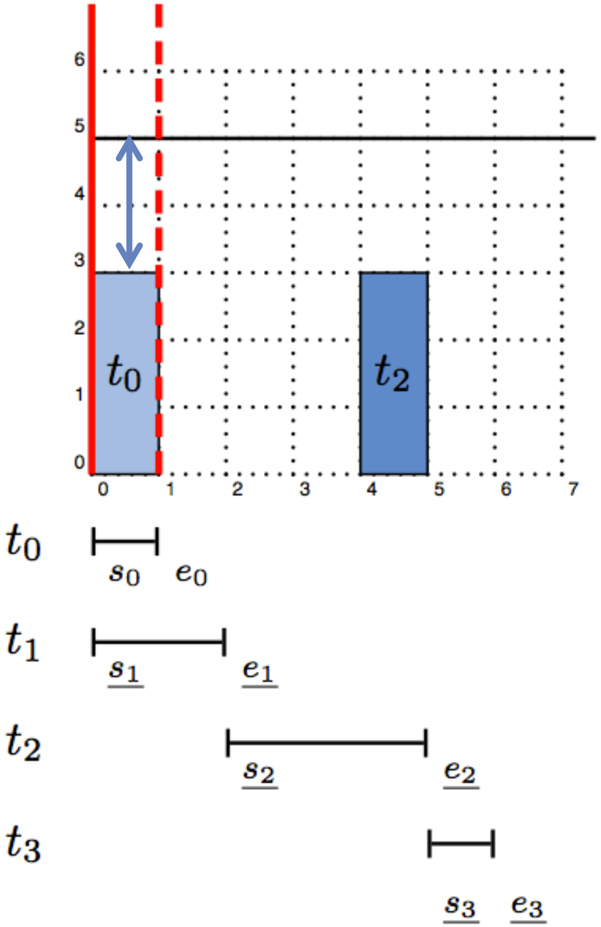
THE DYNAMIC SWEEP ALGORITHM
(PRINCIPLE: DATA STRUCTURES)

To partially avoid rescanning of all tasks:

- **A heap $h_{conflict}$ storing tasks in conflict with the current sweep interval. Tasks are ordered by increasing height.**

- **A heap h_{check} storing tasks not in conflict on the current sweep interval and for which the earliest start is not yet found. Tasks are ordered by decreasing height.**

THE DYNAMIC SWEEP ALGORITHM
(ILLUSTRATION)



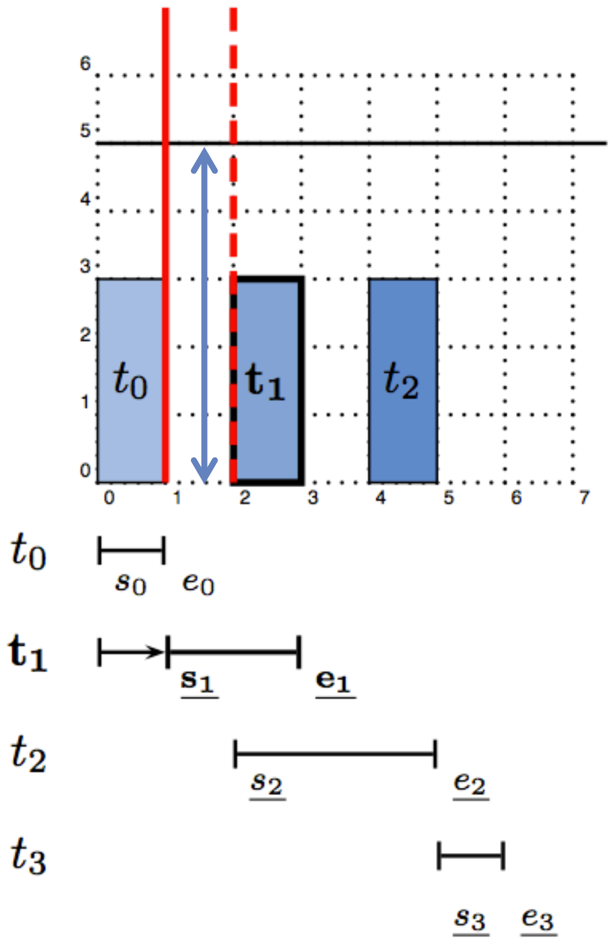
sweep interval = [0,1)

available resource = 2

h_1 (=3) **is greater than** the available resource (=2).

t_1 is added into $h_{conflict}$.

THE DYNAMIC SWEEP ALGORITHM
(ILLUSTRATION)

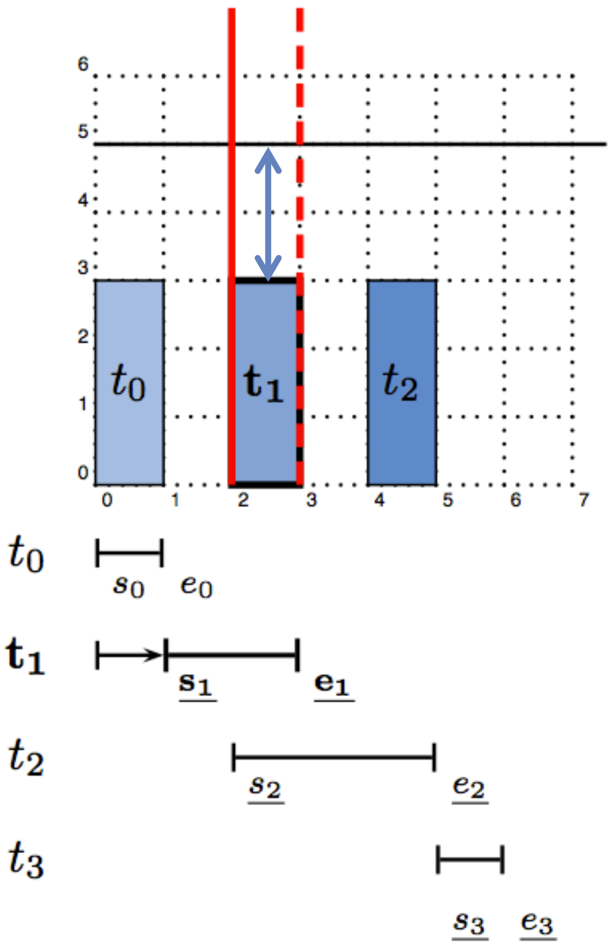


sweep interval = [1,2)
available resource = 5

Top task of $h_{conflict}$ (t_1) **is not greater than** the available resource. Consequently t_1 is removed from $h_{conflict}$ and added into h_{check}.

Earliest start of t_1 is adjusted to 1.
Its conditional event is transformed into 2 events reflecting its new compulsory part

THE DYNAMIC SWEEP ALGORITHM
(ILLUSTRATION)



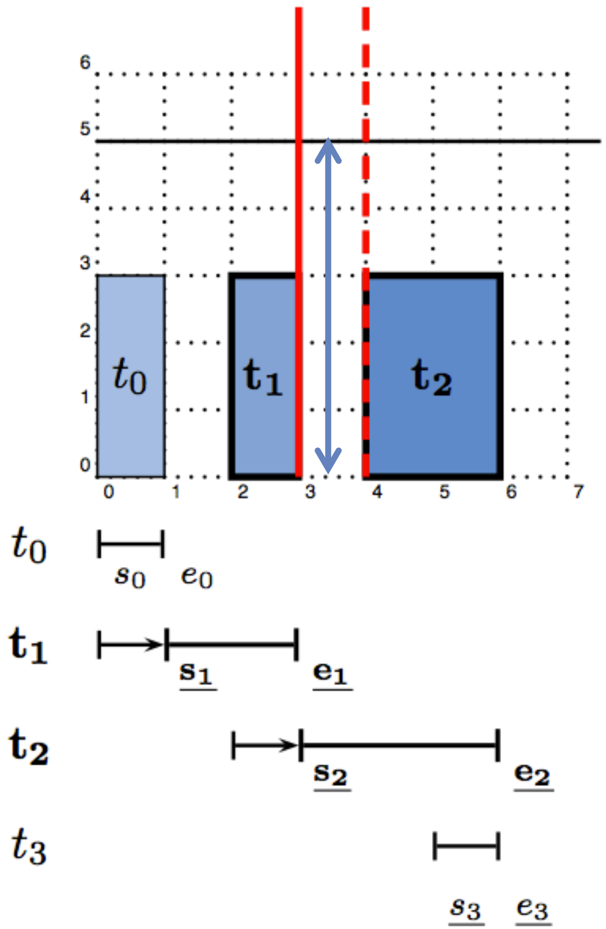
sweep interval = [2,3)

available resource = 2

h_2 (=3) **is greater than** the available resource (=2).

t_2 is added into $h_{conflict}$.

THE DYNAMIC SWEEP ALGORITHM
(ILLUSTRATION)



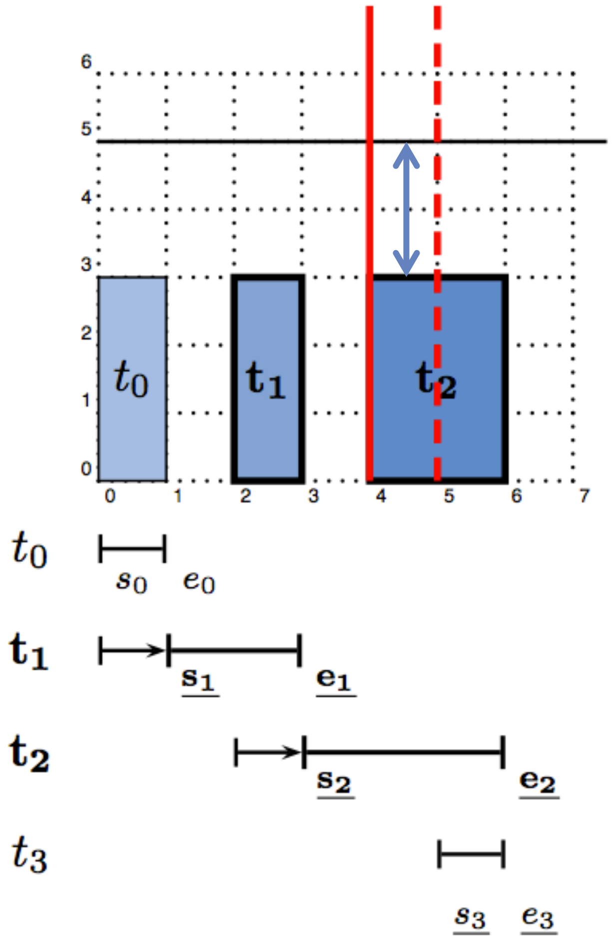
sweep interval = [3,4)

available resource = 5

Top task of $h_{conflict}$ (t_2) **is not greater than** the available resource. Consequently t_2 is removed from $h_{conflict}$ and added into h_{check}.

Earliest start of t_2 is adjusted to 3.
Event related to its end of compulsory part is pushed from 5 to 6.

THE DYNAMIC SWEEP ALGORITHM
(ILLUSTRATION)

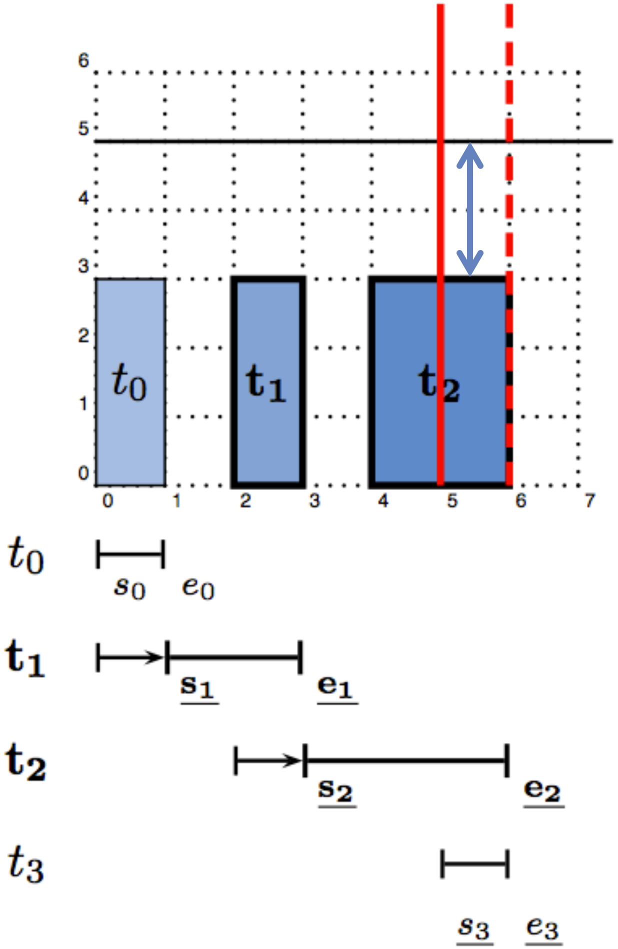


sweep interval = [4,5)

available resource = 2

Nothing to do.

THE DYNAMIC SWEEP ALGORITHM
(ILLUSTRATION)



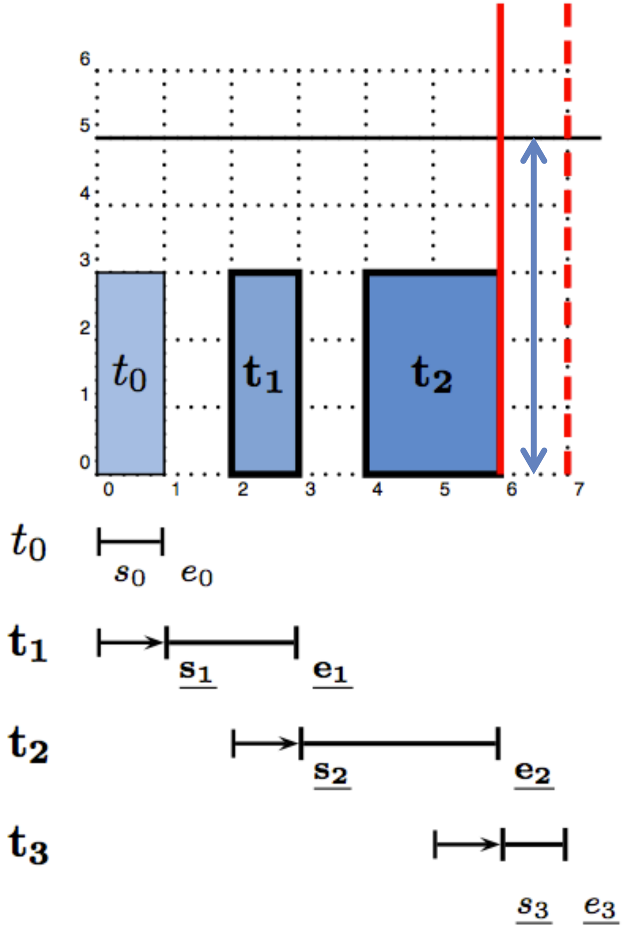
sweep interval = [5,6)

available resource = 2

h_3 (=3) **is greater than** the available resource (=2).

t_3 is added into $h_{conflict}$.

THE DYNAMIC SWEEP ALGORITHM
(ILLUSTRATION)



sweep interval = [6,7)

available resource = 5

Top task of $h_{conflict}$ (t_3) **is not greater than** the available resource. Consequently t_3 is removed from $h_{conflict}$.

Earliest start of t_3 is adjusted to 6.
Nothing else to do.

THE DYNAMIC SWEEP ALGORITHM
(PROPERTY AND COMPLEXITY)

- **A worst-case time complexity of *$O(n^2 \log n)$* where *n* is the number of tasks.**

  There is a variant with a worst-case time complexity of $O(n^2)$,
  but the $O(n^2 \log n)$ version scales better.

- **Property after a call to *sweep_min:***

For any task *t* in *T*, one can schedule *t* at its earliest start without exceeding the resource limit wrt. the cumulated profile of *T\{t}*.

THE DYNAMIC SWEEP ALGORITHM
(GREEDY MODE USING FILTERING)

Why ?

To handle larger (10 million tasks) instances in a CP solver.

How ?

It reuses the *sweep_min* part but directly fixes the start of the task rather than adjusting it. Then, the sweep-line is **reset to this start** and the process continues until all tasks get fixed or a resource overflow occurs.

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KEY IDEAS

Handle all cumulative and precedence in one pass (adjusting min)

Reuse the idea used just for a set of precedence (topological ordering) but also consider the cumulative constraints while propagating

Each constraint is not propagated in isolation (i.e. independently from the other) but gradually

1. Sort the variables wrt their min,
2. Propagate all constraints just on the first variable (unless its min increases and it is not the first variable anymore)
3. Discard first variable and continue …

EVENTS TYPES

(SCP,t,latest start): Start of Compulsory part (even if no CP)

(ECPD,t,earliest end): End of Compulsory part (only if CP)

(PR,t,earliest start): Pruning (only if not fixed)

(RS,t,earliest end): Release Successors (at least one successor)

prevent the earliest starts of the successor of t from being adjusted before the final earliest start of task t has been determined

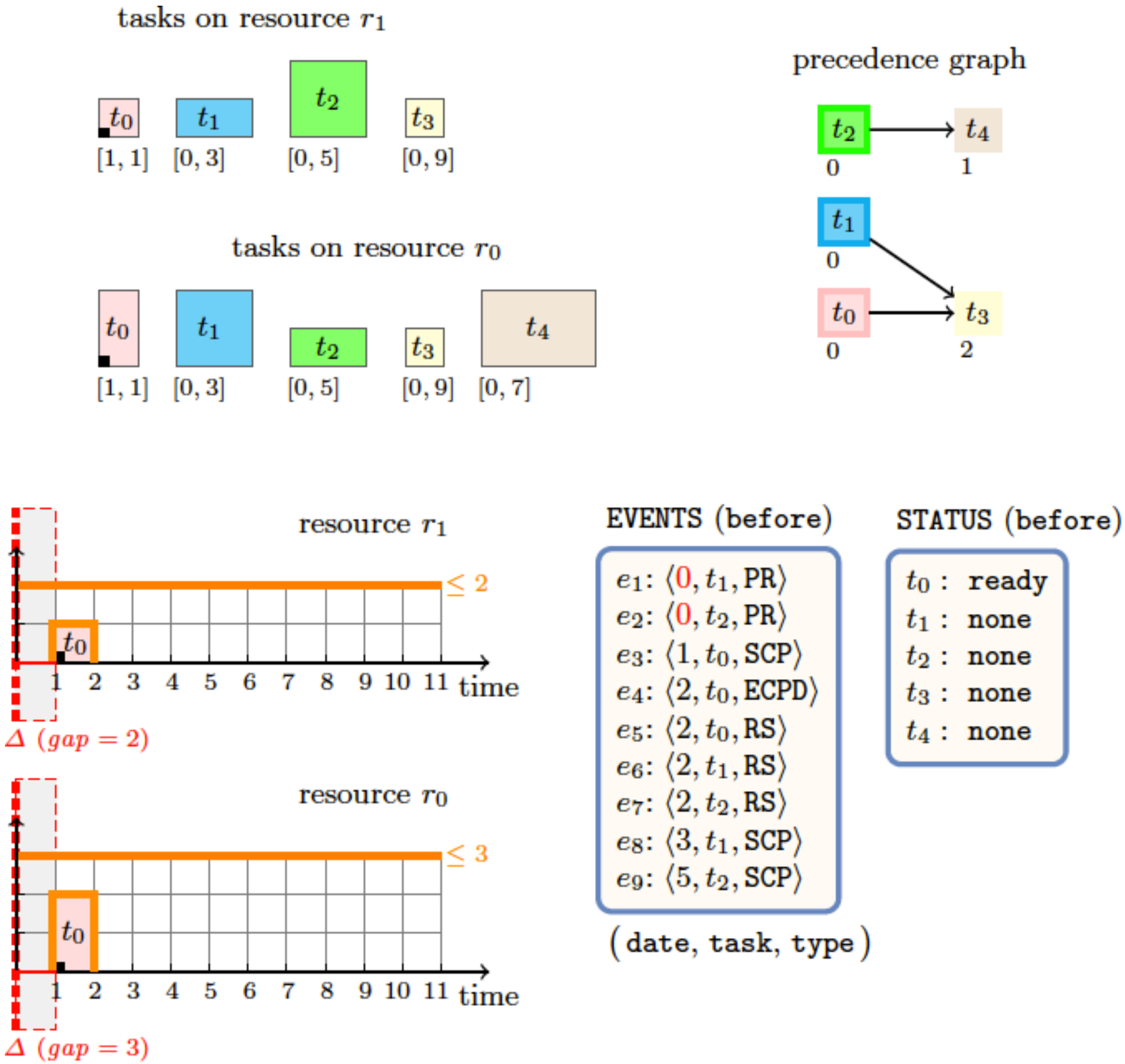

Fig. 1: initialization and processing events $\langle 0, t_1, \mathrm{PR} \rangle$ and $\langle 0, t_2, \mathrm{PR} \rangle$

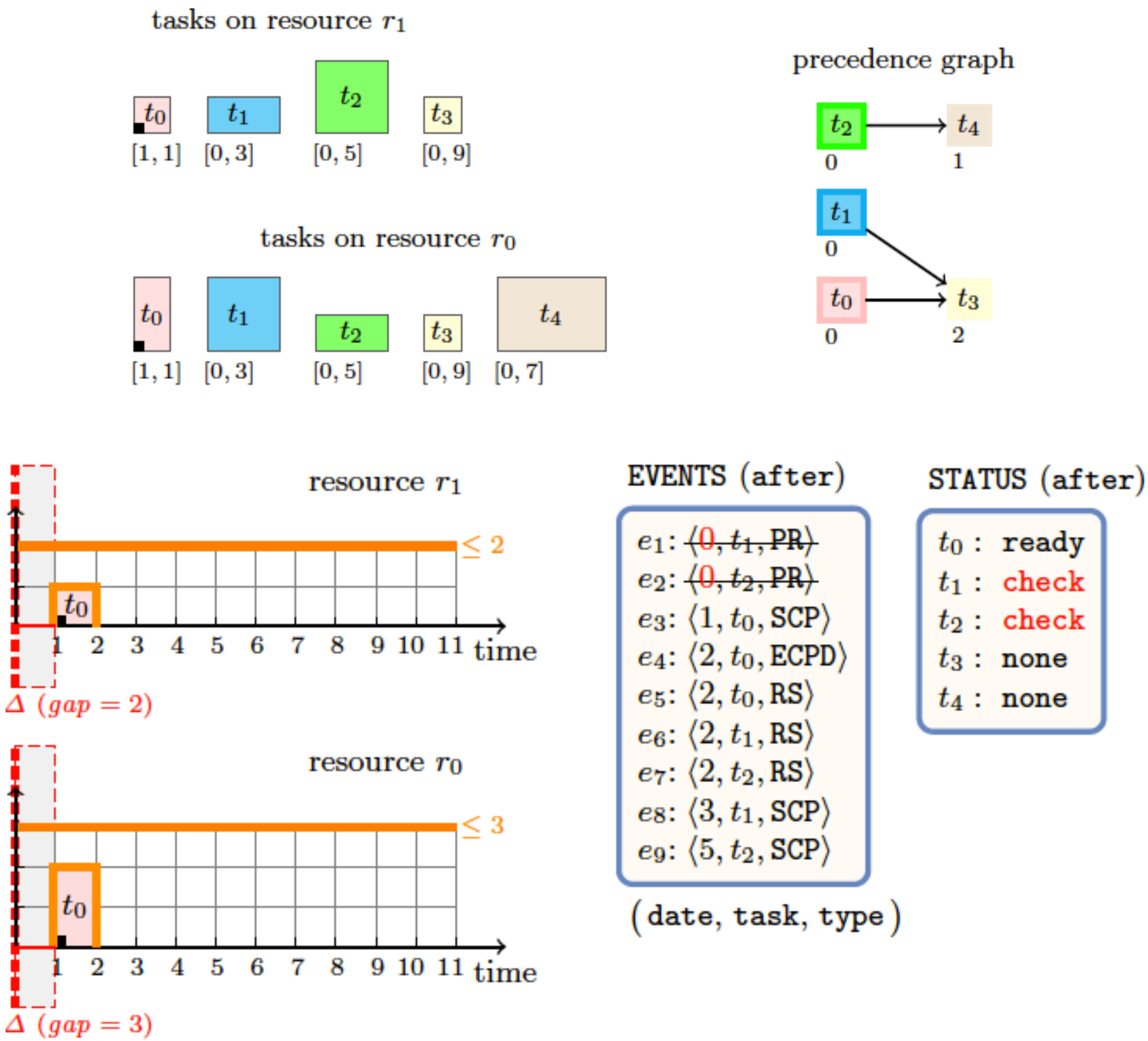

Fig. 2: after initialization and processing events $\langle 0, t_1, \mathrm{PR} \rangle$ and $\langle 0, t_2, \mathrm{PR} \rangle$

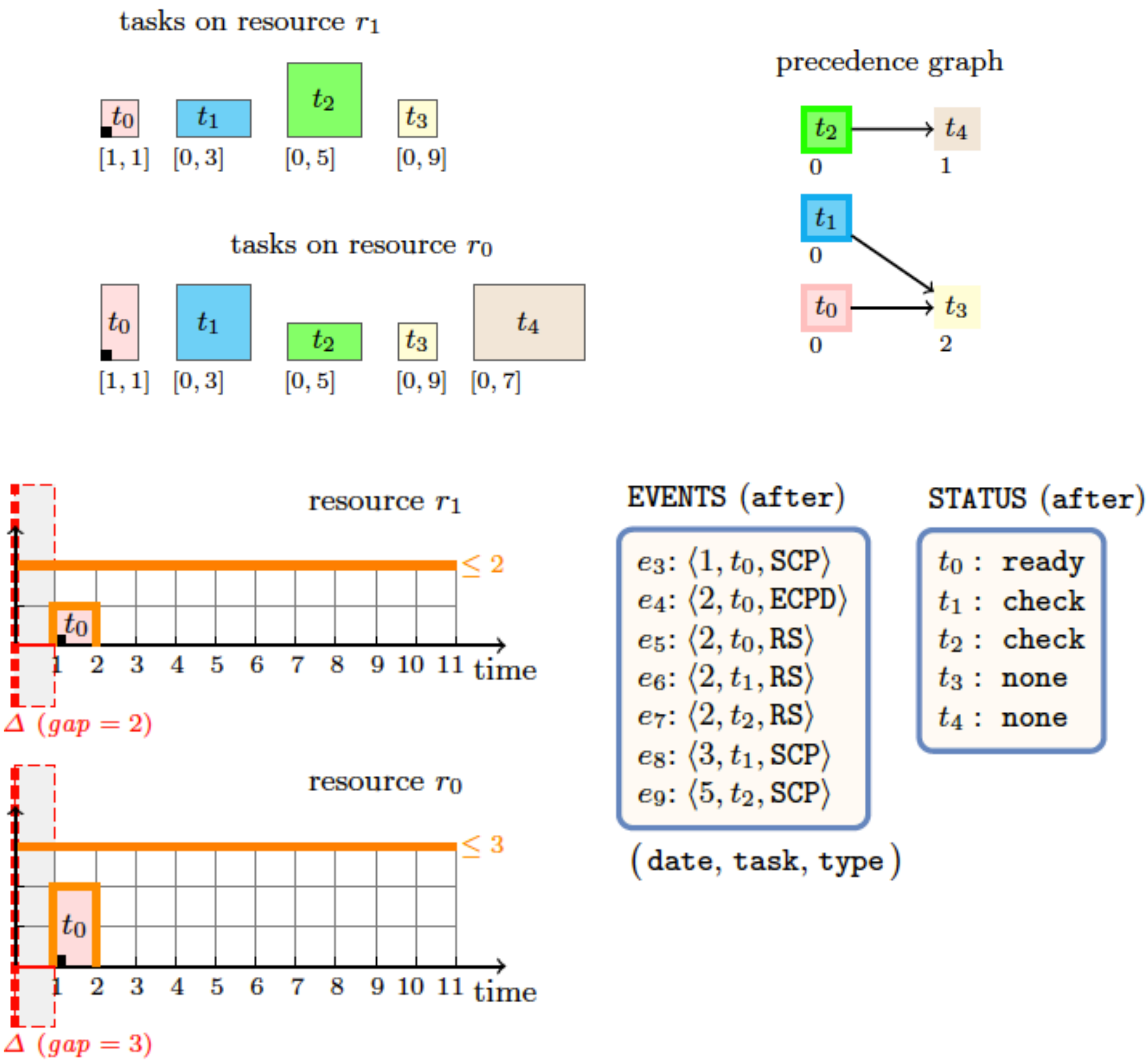

Fig. 3: filter min wrt $[\Delta = 0, \Delta_{next} = 1[$: no pruning

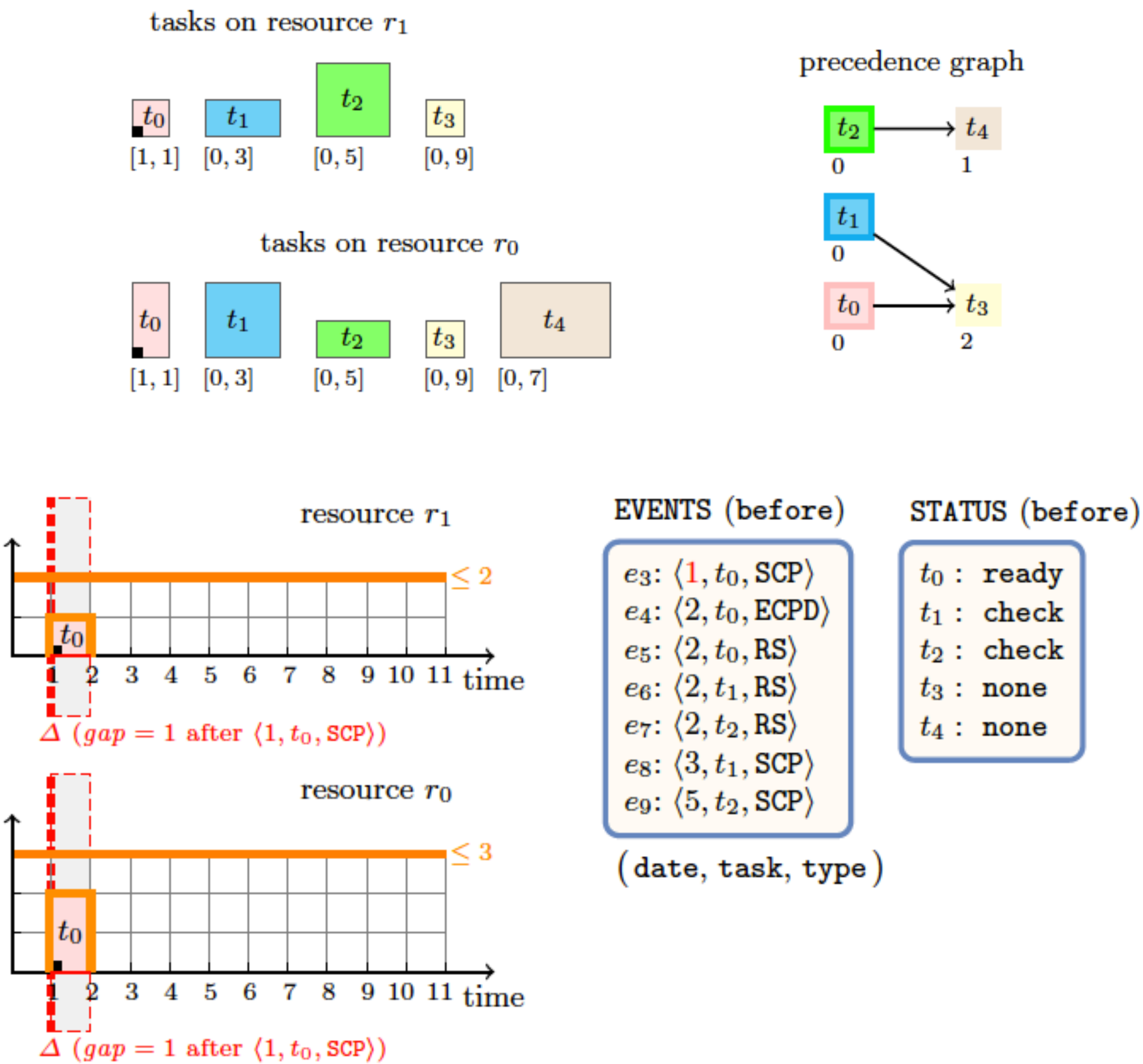

Fig. 4: process event $\langle 1, t_0, \mathrm{SCP}\rangle$

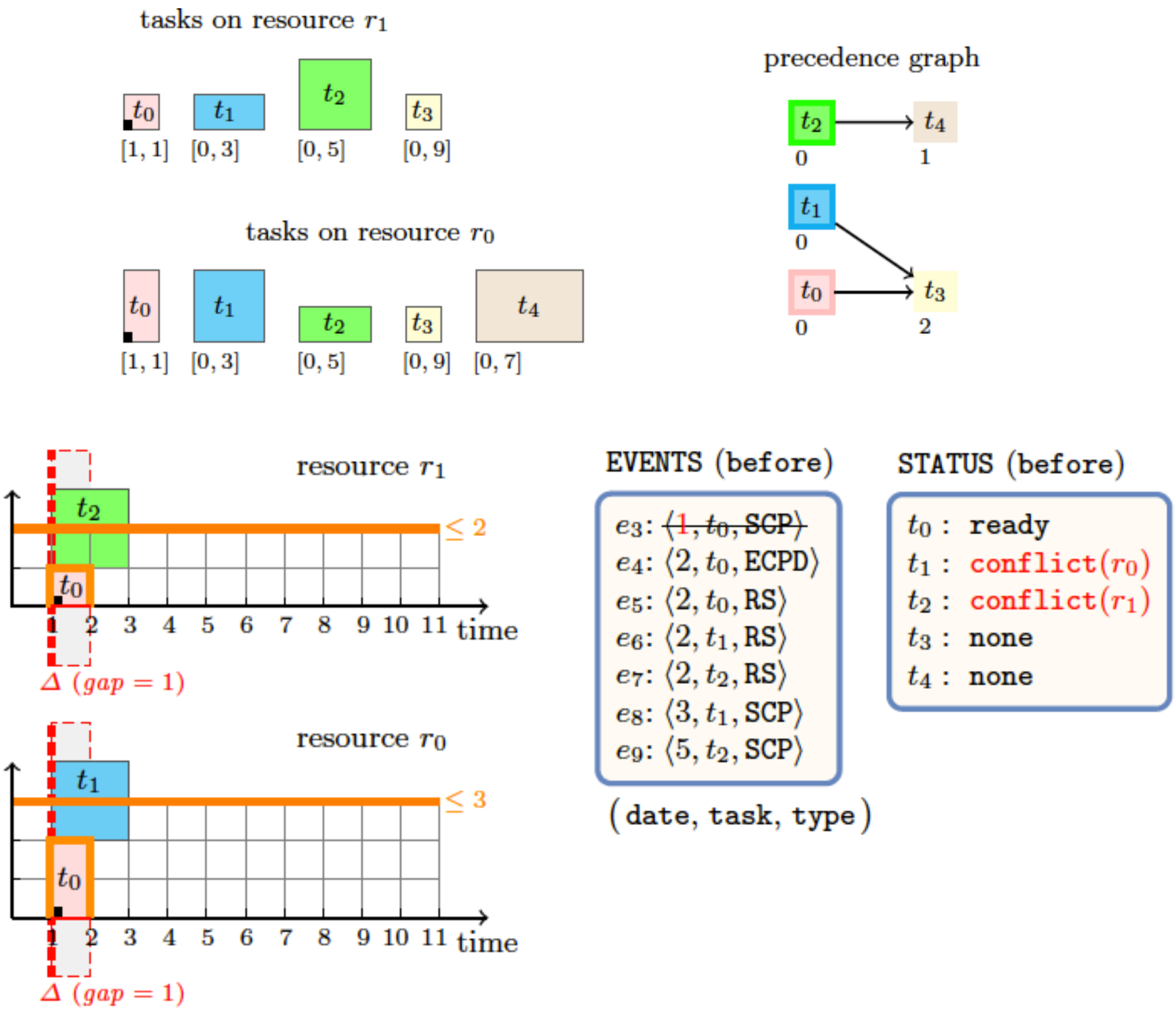

Fig. 5: filter min wrt $[\Delta = 1, \Delta_{next} = 2[$ after processing event $\langle 1, t_0, \mathrm{SCP} \rangle$

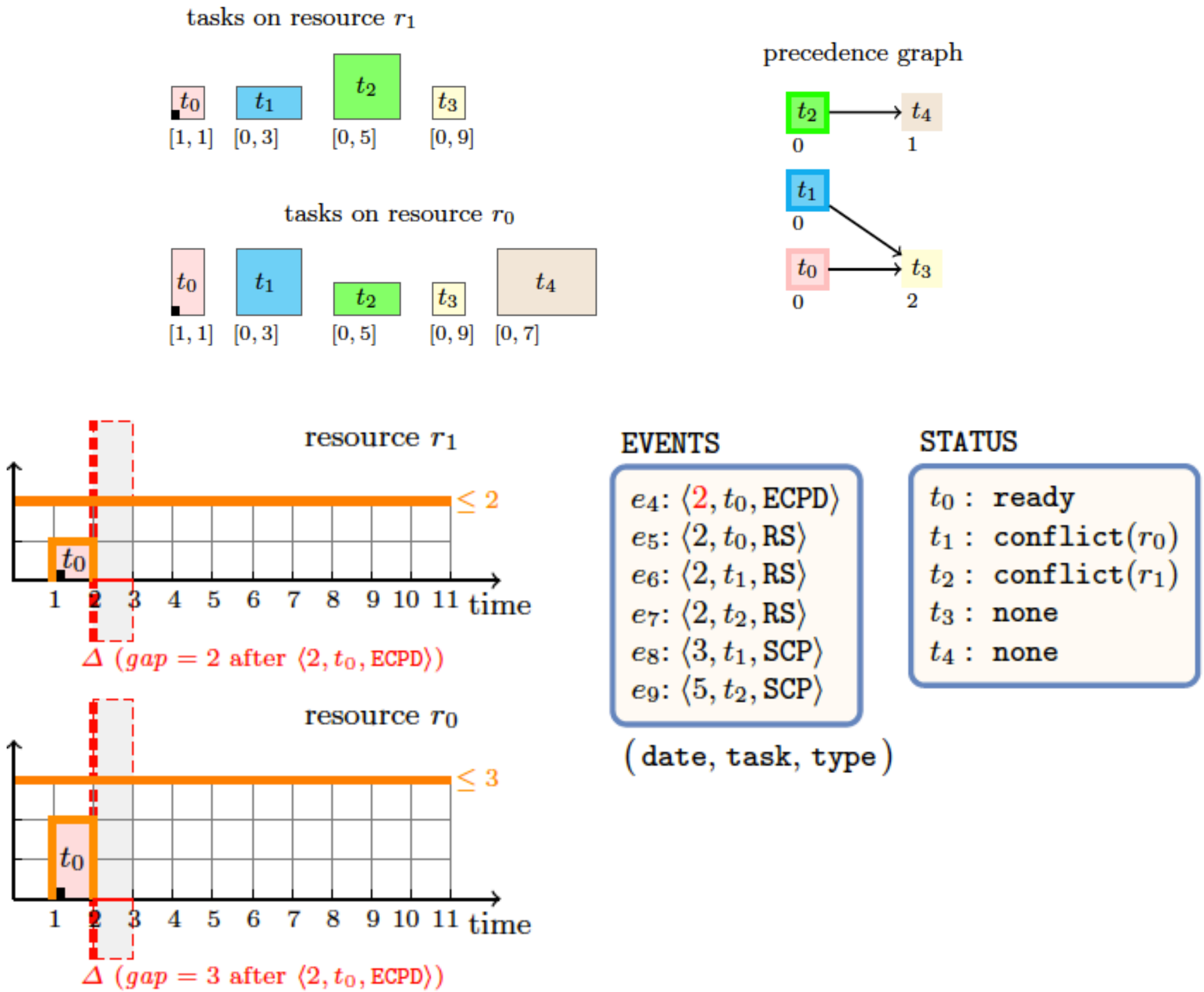

Fig. 6: process event $\langle 2, t_0, \text{ECPD}\rangle$ (gaps increase since end of compulsory part)

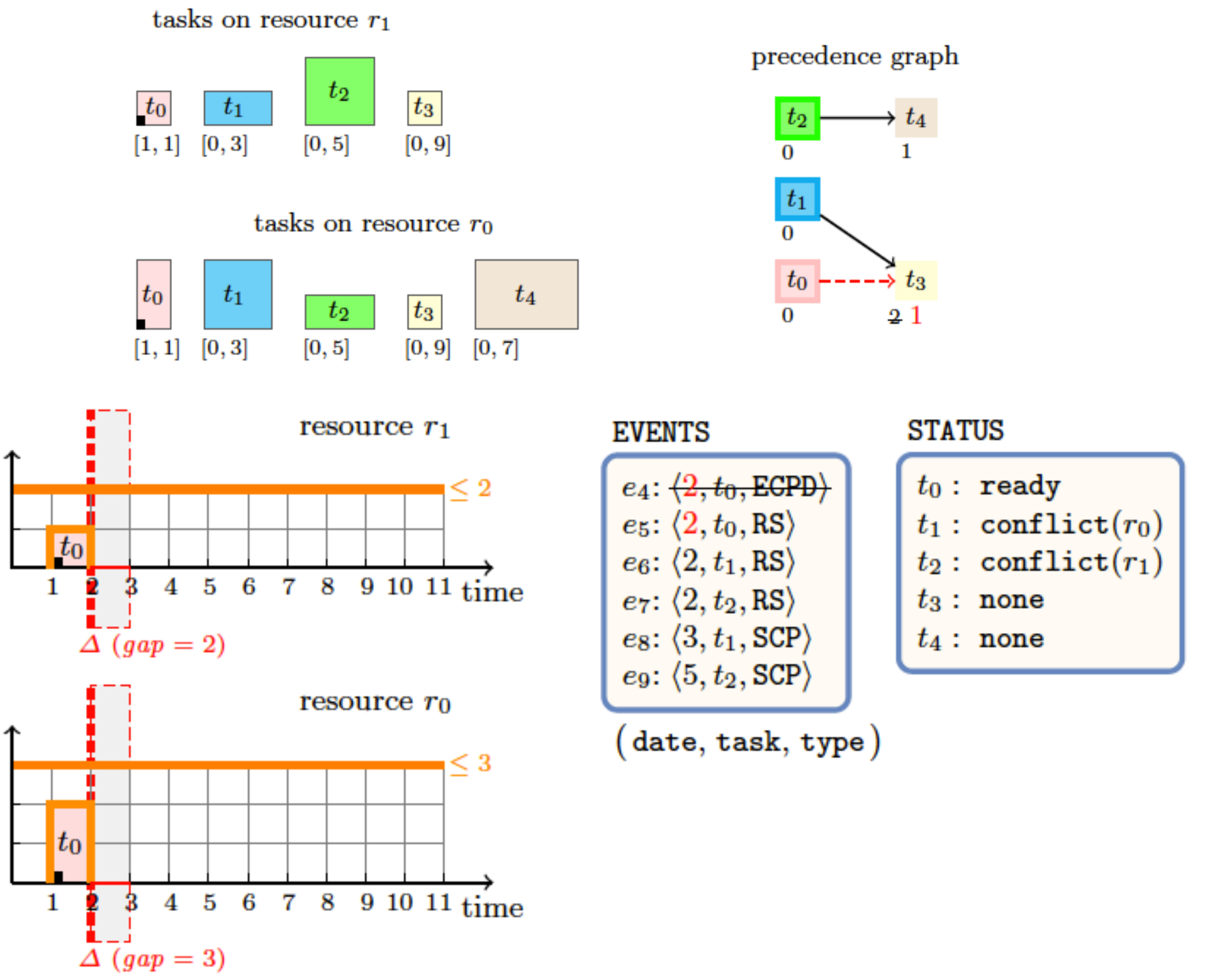

Fig. 7: process event $\langle 2, t_0, \text{RS} \rangle$ (t_0 has reached its final earliest end)

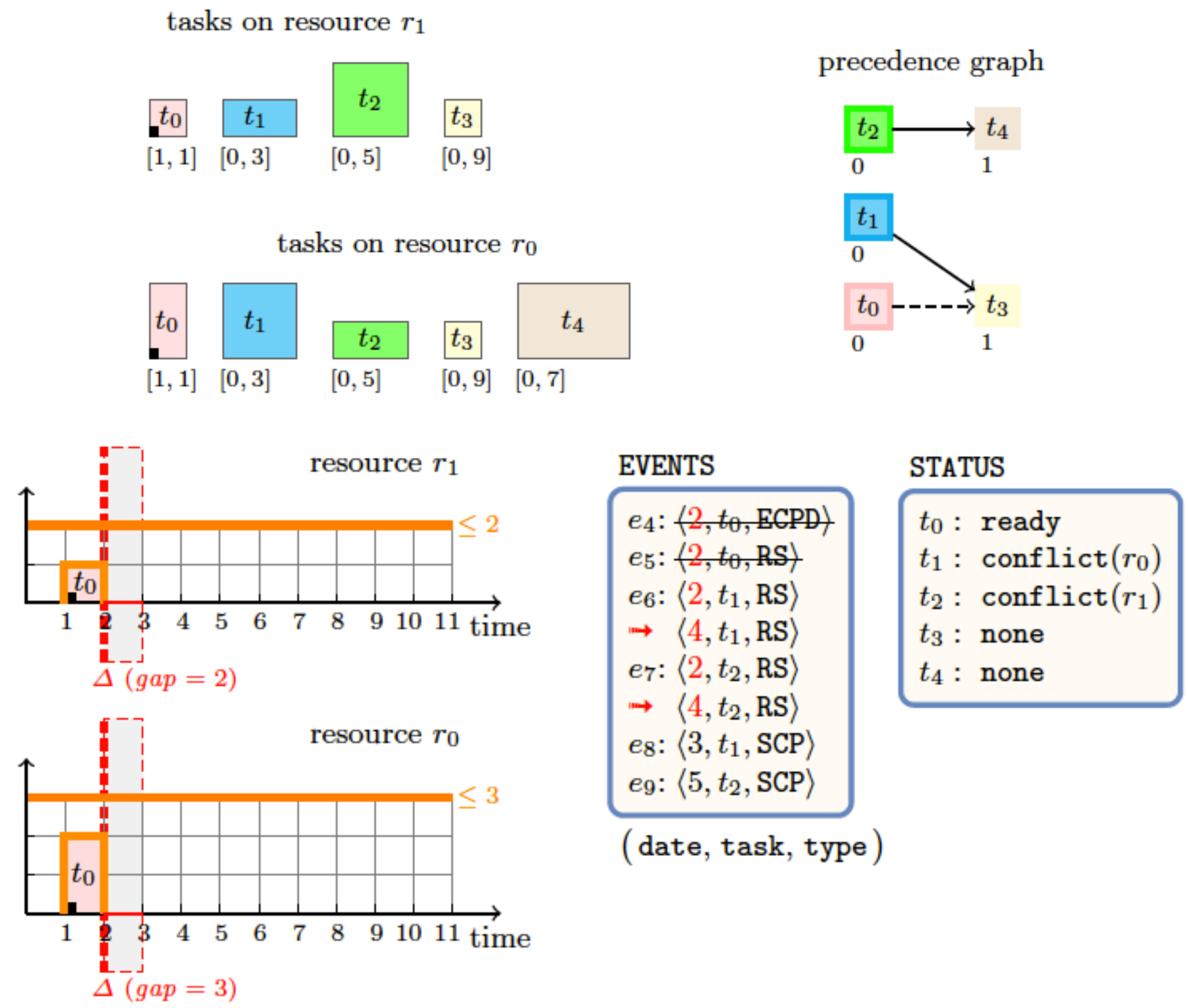

Fig. 8: process events $\langle 2, t_1, \text{RS} \rangle$, $\langle 2, t_2, \text{RS} \rangle$ (t_1, t_2 do not have yet reached their final earliest end, so both events are shifted)

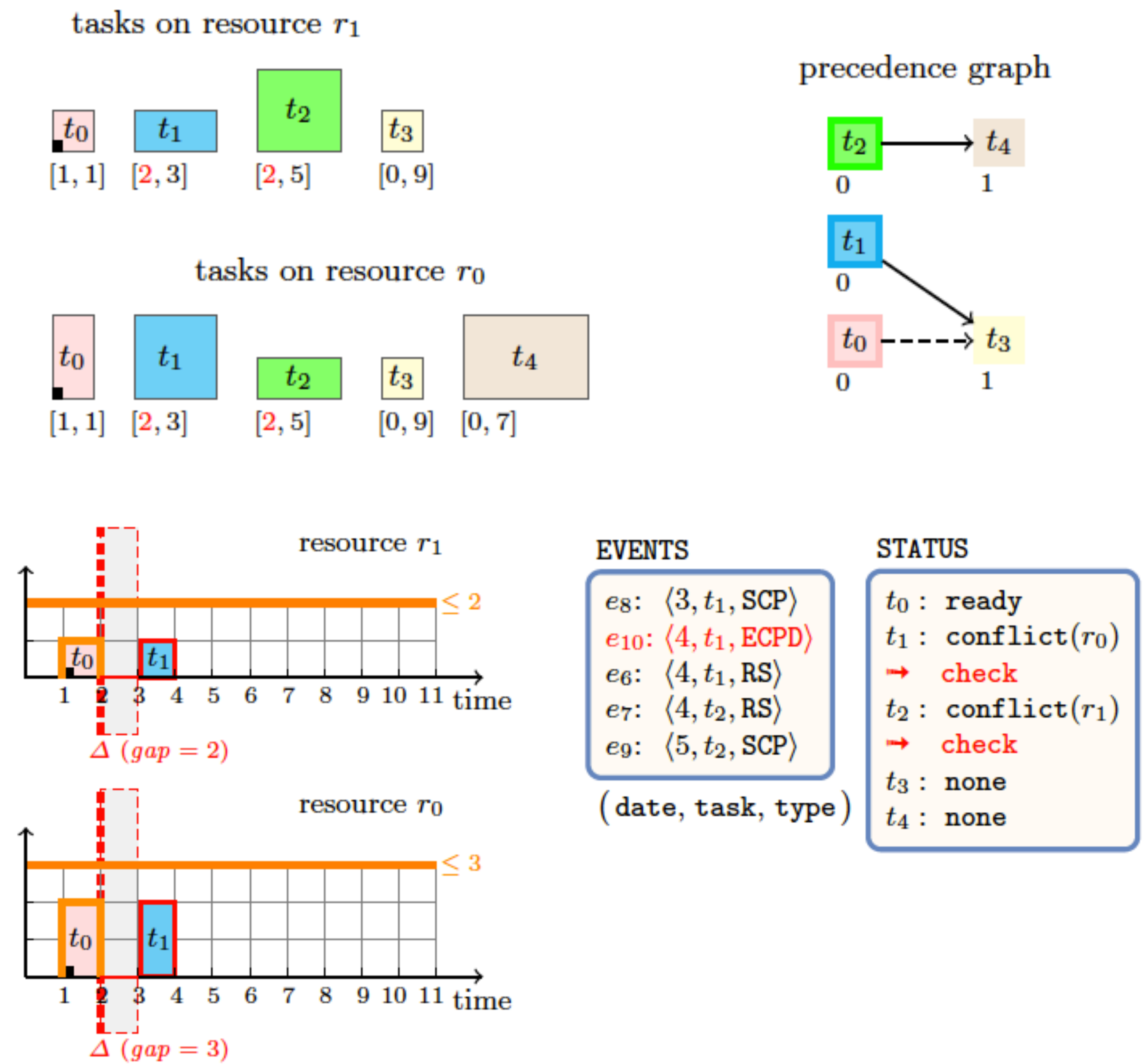

Fig. 9: filter min wrt $[\Delta = 2, \Delta_{next} = 3[$ after processing events at date 2 (create compulsory part for t_1 and corresponding event for end of compulsory part)

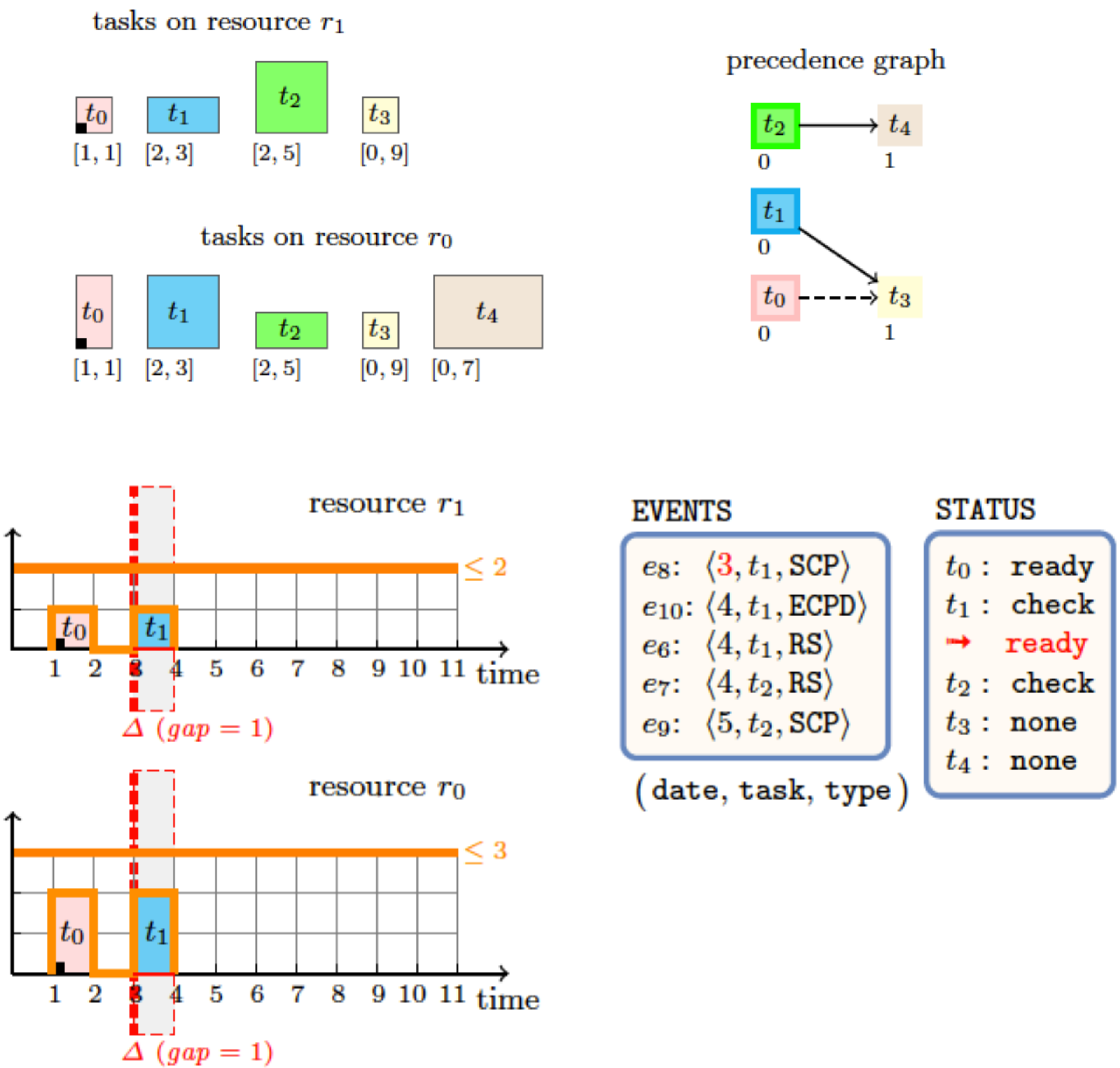

Fig. 10: process event $\langle 3, t_1, \text{SCP}\rangle$ (find final earliest start for t_1)

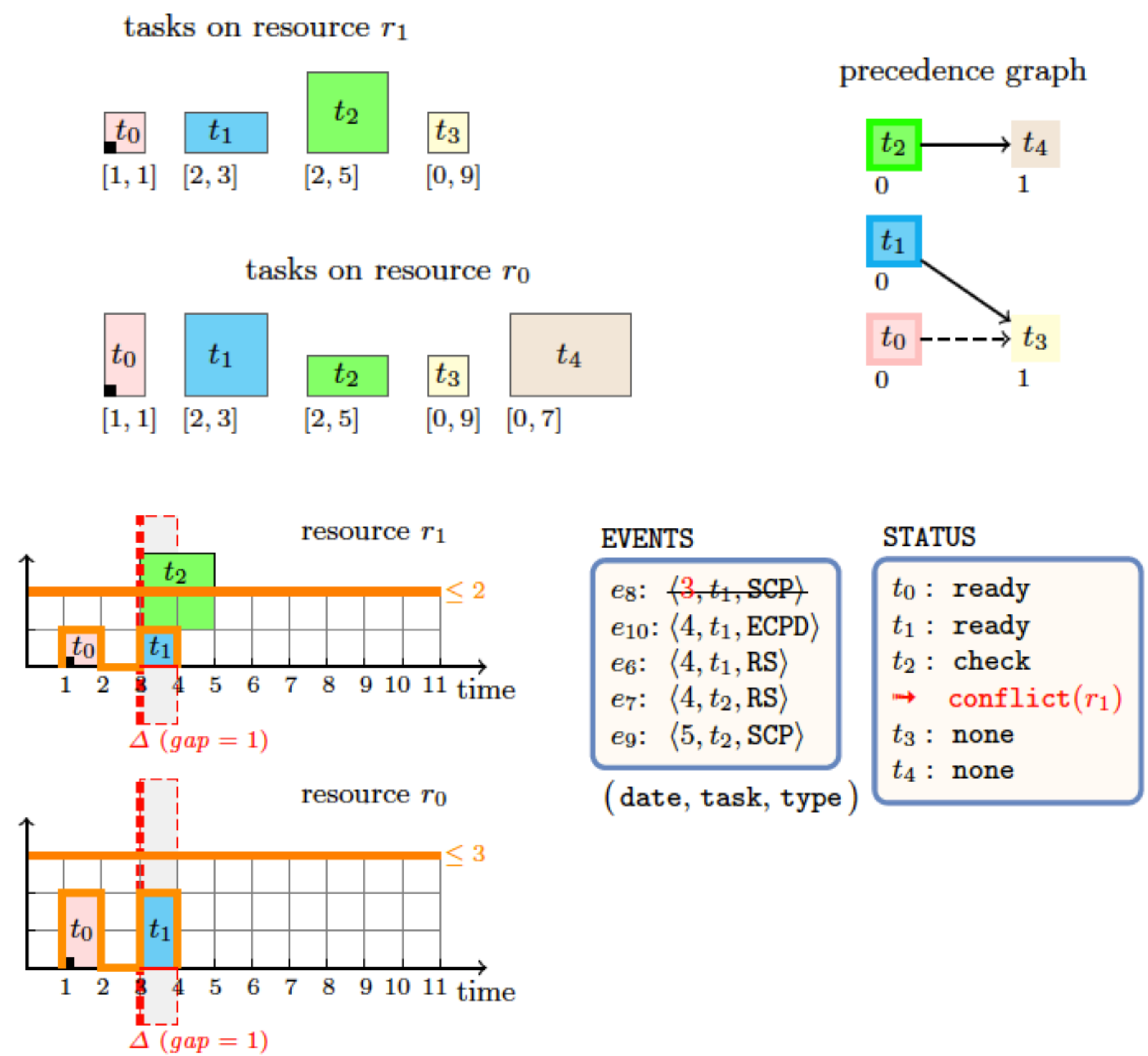

Fig. 11: filter min wrt $[\Delta = 3, \Delta_{next} = 4[$ after processing events at date 3 $(\langle 3, t_1, \mathsf{SCP} \rangle)$

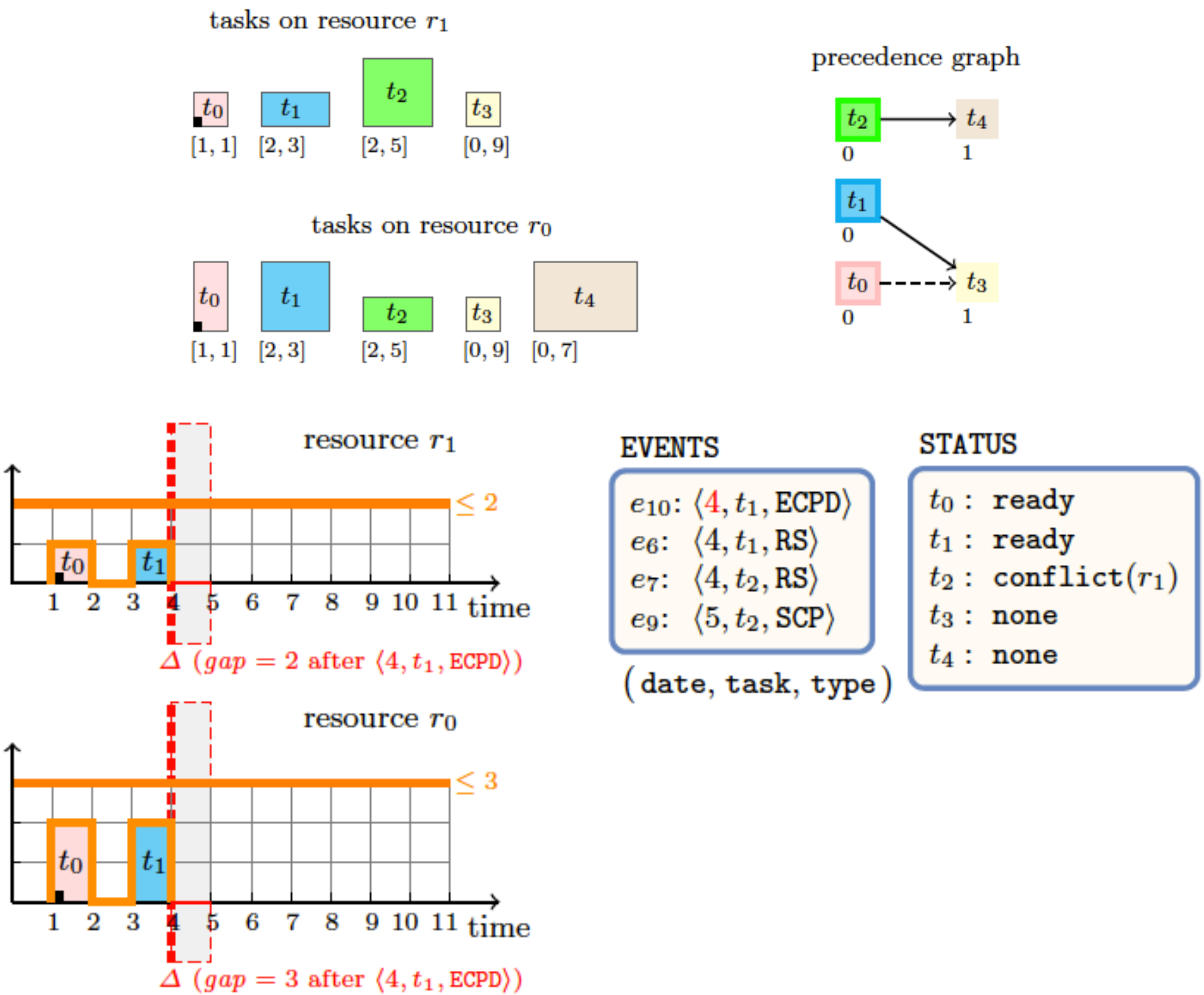

Fig. 12: process event $\langle 4, t_1, \mathsf{ECPD}\rangle$ (gaps increase since end of compulsory part)

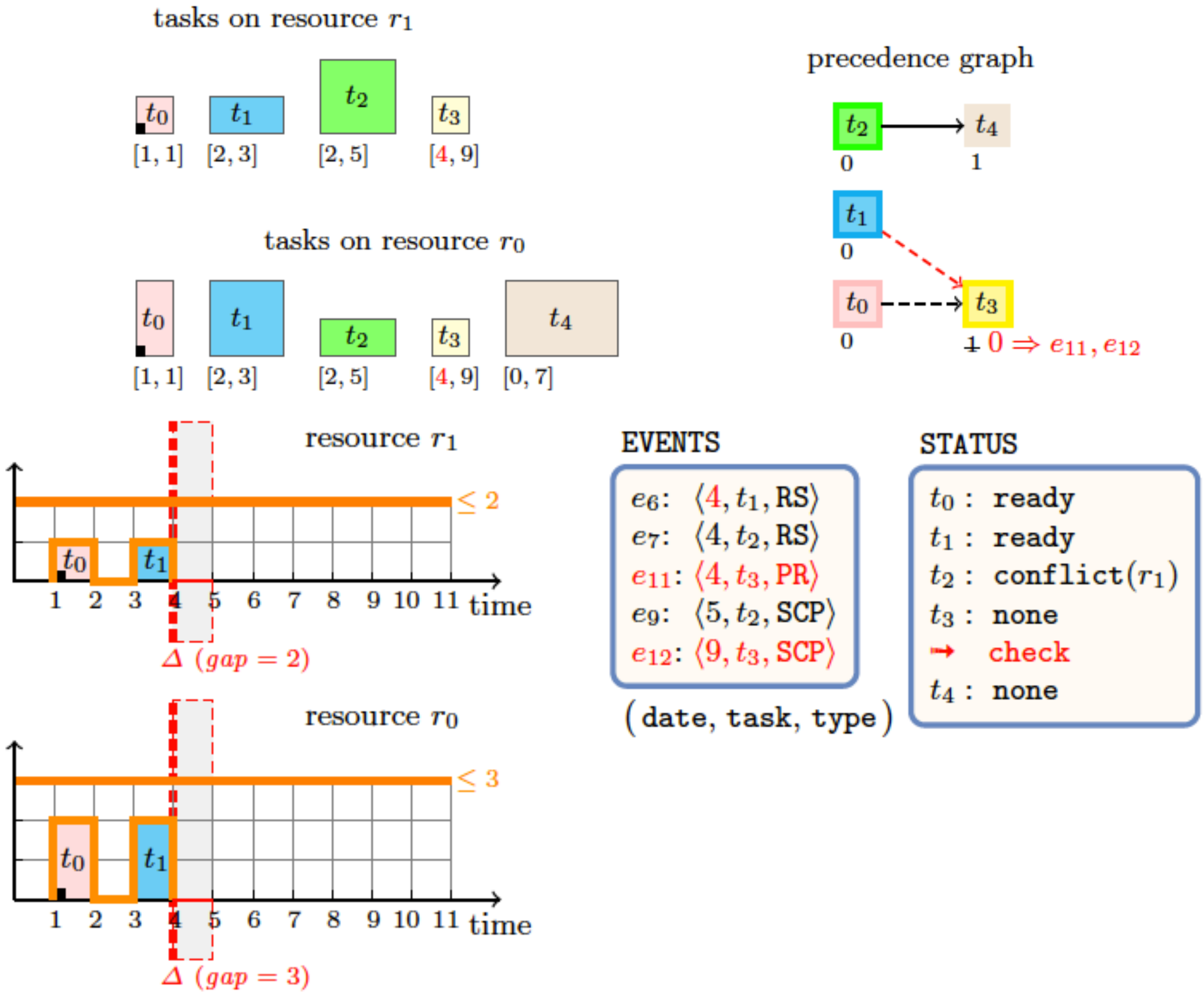

Fig. 13: process event $\langle 4, t_1, \text{RS} \rangle$ (t_1 has reached its final earliest end, its successor

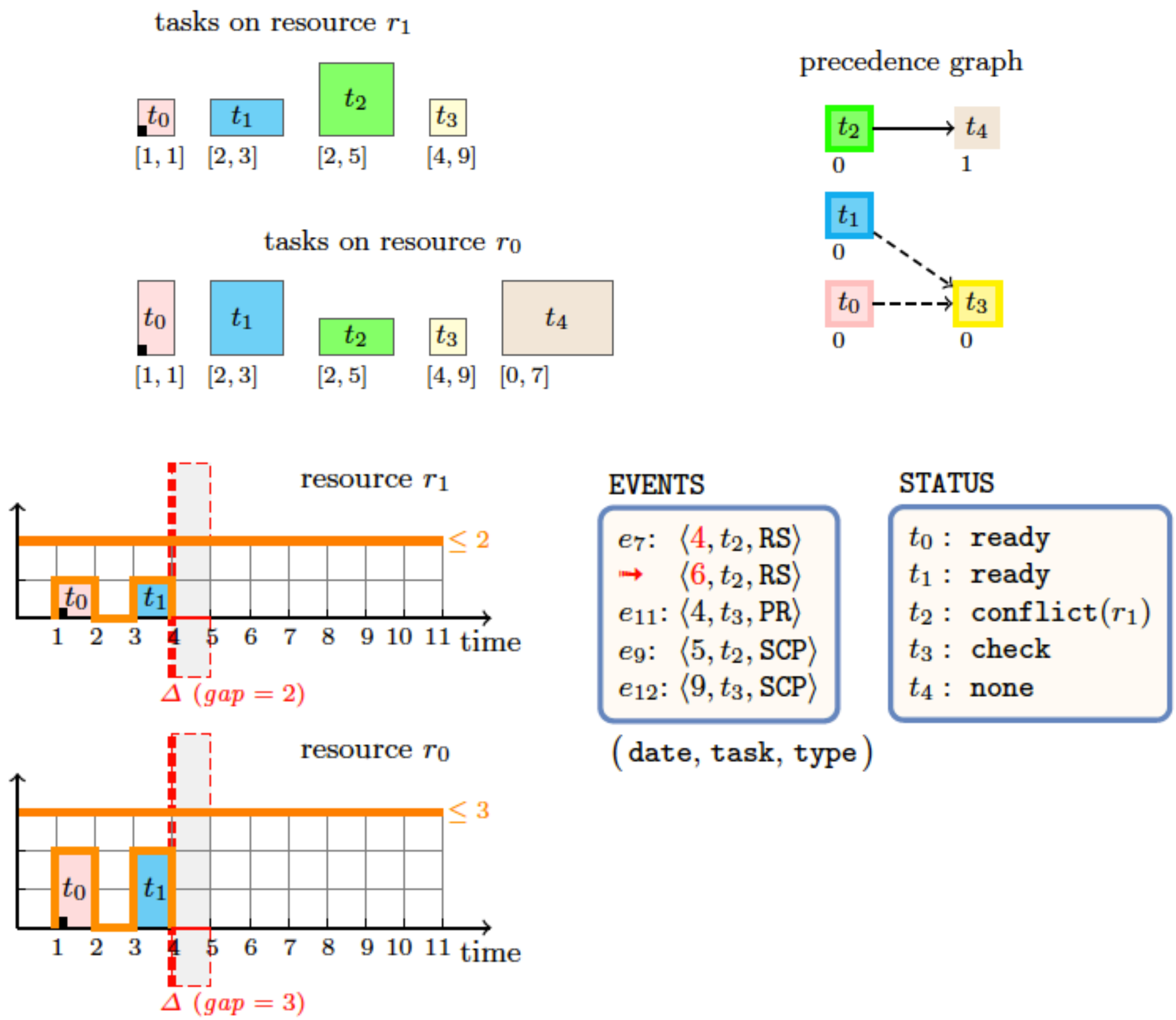

Fig. 14: process event $\langle 4, t_2, \mathsf{RS}\rangle$ (t_2 has not yet reached its final earliest end, consequently shift the RS event)

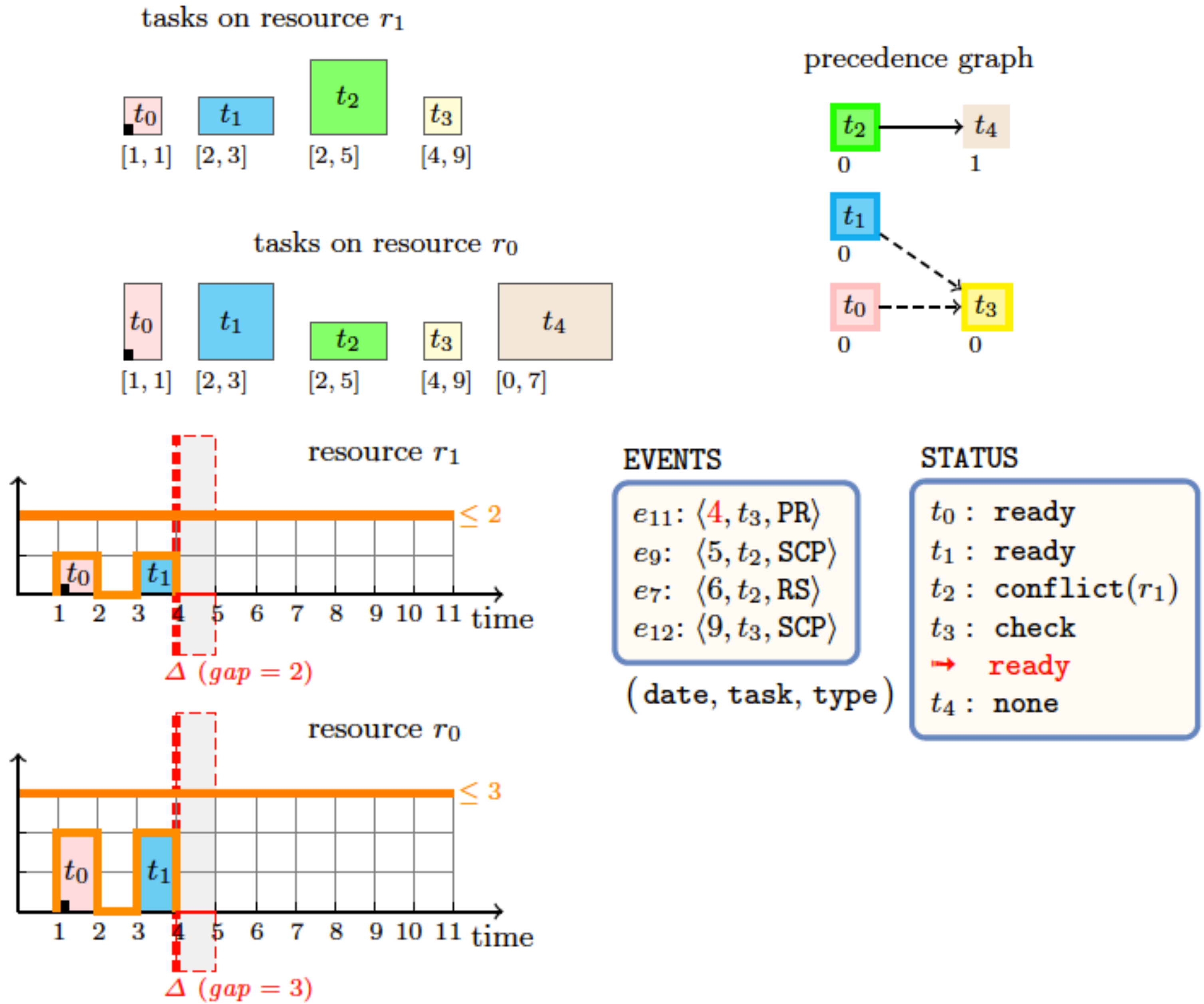

Fig. 15: process event $\langle 4, t_3, \mathrm{PR}\rangle$ (t_3 can be include in $[\Delta = 4, \Delta_{next} = 5[$ so t_3 has reached its final earliest start)

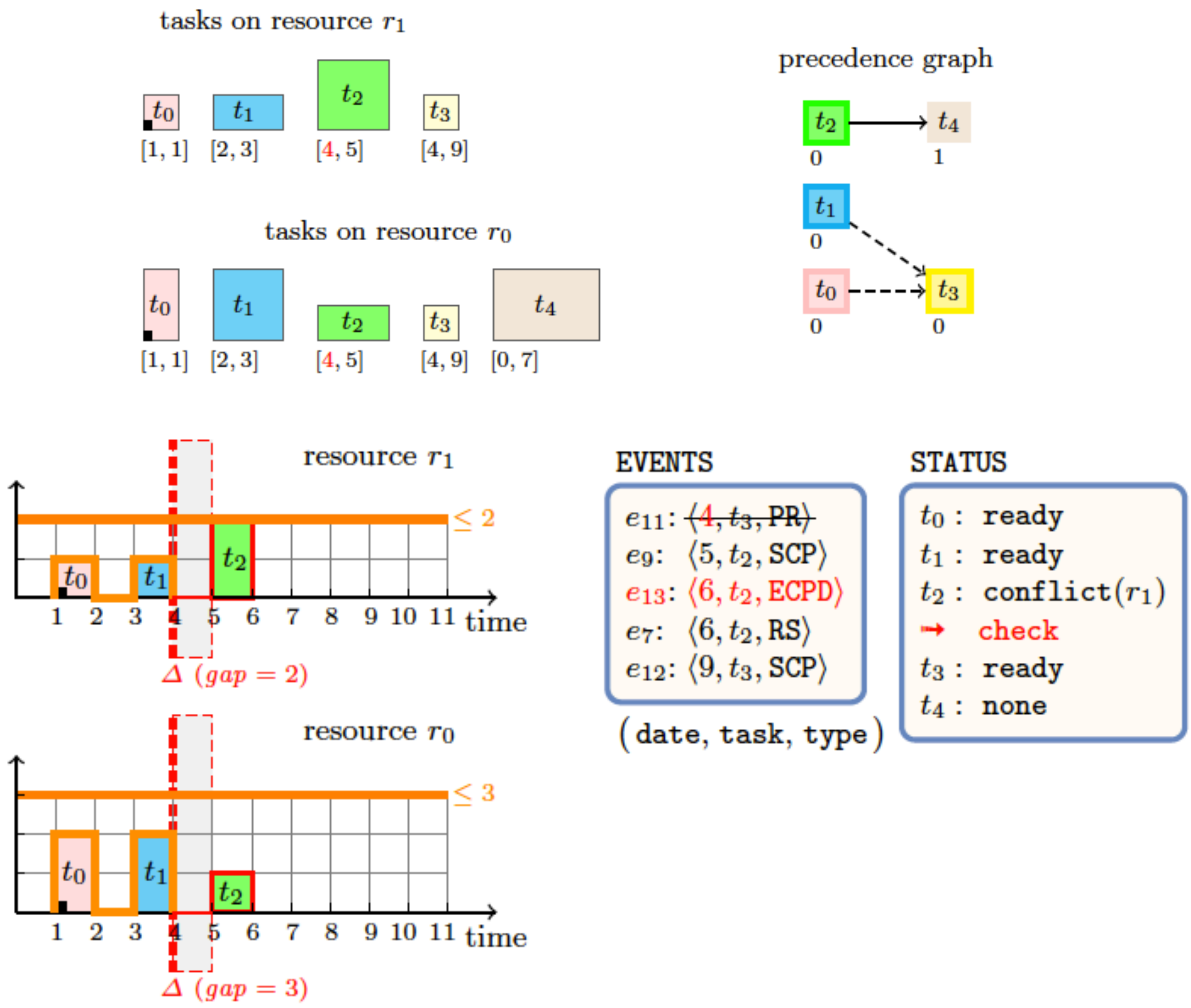

Fig. 16: filter min wrt $[\Delta = 4, \Delta_{next} = 5[$ after processing events at date 4 (create compulsory part for t_2 and corresponding event for end of compulsory part)

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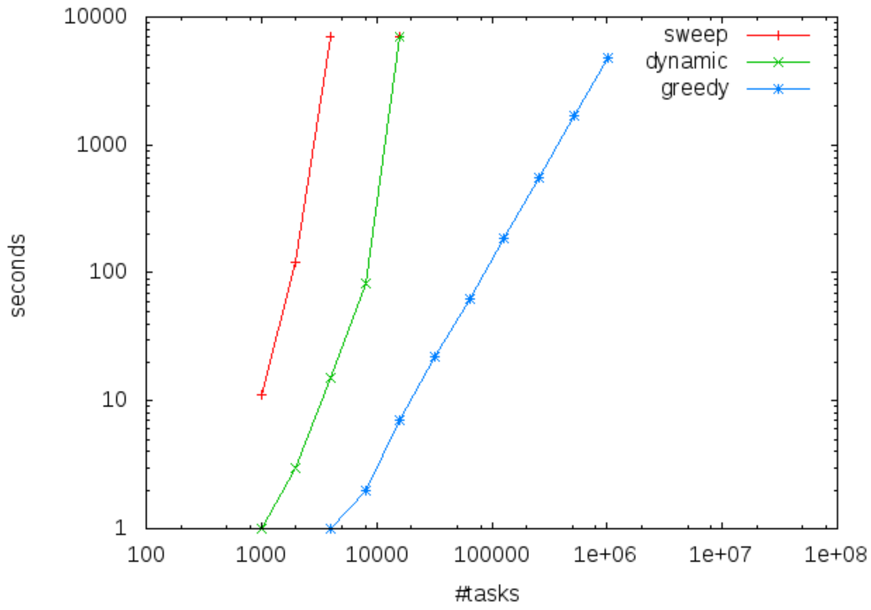
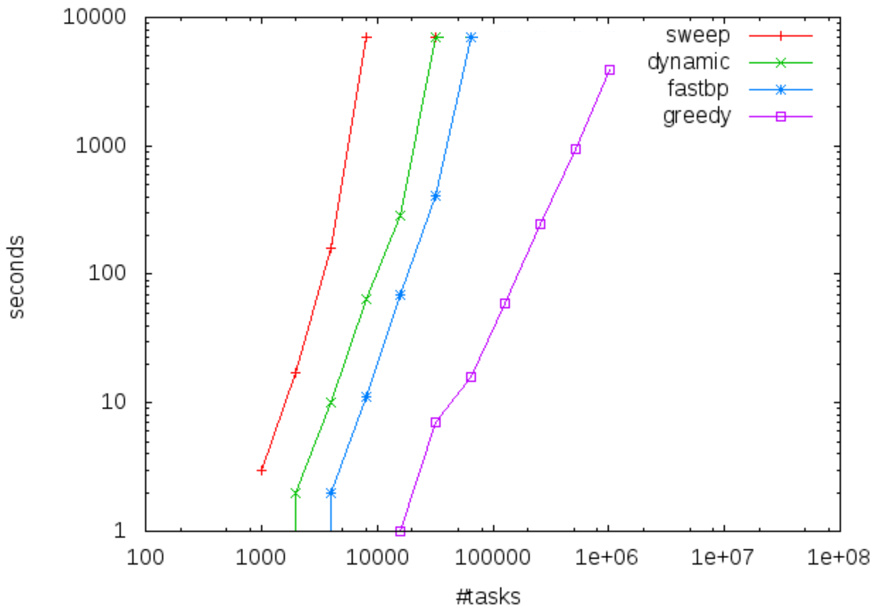
EVALUATION

Cumulative



Bin-Packing



- Random instances with a density close to 0.7.
- A **speedup increasing** with the number of tasks.
- The dynamic is **more robust** than the 2001 sweep wrt. different heuristics.
- The greedy mode could handle:

  1 million tasks in **12 minutes**

  up to **10^7 tasks** in ~8h (swap).

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CONCLUSION

- a **lean** sweep based filtering algorithm
- **dynamically** handle creation/extension of CP
- **faster** and **more scalable** than the 2001 sweep
- handle up to **10 million tasks** in greedy mode.

  handle up to **1 million tasks** with **2 millions precedences** and **64 resources** in greedy mode.

MORE INFORMATION

CP 2012 paper:

One single cumulative constraint,

LNCS vol. 7514, pp. 439-454

CPAIOR 2013 paper:

several cumulative constraints

Available at http://www.cis.cornell.edu/ics/cpaior2013/papers.php

Technical report (April 2013)

Several cumulative + precedence constraints

Available at http://soda.swedish-ict.se/5504/