Non-symmetric linear solvers based on multipreconditioning

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Krylov subspace methods [5] are powerful tools for solving linear systems of the form

$$\boldsymbol{A}\boldsymbol{x} = \boldsymbol{b}; \text{ with } \boldsymbol{A} \in \mathbb{R}^{n \times n} \text{ non-singular and } \boldsymbol{b} \in \mathbb{R}^n.$$

For many applications, they are known to converge very slowly, often after a long stagnation. A natural way to fix this is by preconditioning the linear system, *i.e.,* by solving $\mathbf{H}\boldsymbol{A}\boldsymbol{x} = \mathbf{H}\boldsymbol{b}$ where $\mathbf{H}$ is a (rather cheap) approximation of $\boldsymbol{A}^{-1}$. Another natural way of accelerating convergence is to enlarge the space in which the approximate solution is optimized at each iteration, *e.g.,* through deflation [3, 7] or block Krylov methods.

Multipreconditioning [2] makes both of these ideas work together: several preconditioners for $\boldsymbol{A}$ are chosen, and they are all applied at every iteration in order to provide an enlarged search space for the approximate solution. For symmetric positive definite problems, Multipreconditioned CG (together with domain decomposition) has proven to reduce significantly the number of iterations needed to achieve convergence. An adaptive version [6] of the algorithm has also been introduced that mixes multipreconditioned and preconditioned iterations with the objective of efficiency (see [1] for a performance study).

For non-symmetric linear systems, multipreconditioned GMRES was first proposed [4], followed by multipreconditioned Orthomin and multipreconditioned BiCG [6]. This proposal aims at further developing non-symmetric multipreconditioning in order to **provide an efficient and robust parallel solver for large order sparse linear systems**. This could have a strong impact in science and engineering.

The objective of the postdoc would be the following.

- Provide a choice of multipreconditioner (possibly from domain decomposition) that achieves the robustness objective.
- Provide an adaptive procedure to decrease memory requirements and time spent in orthogonalization (two known bottlenecks of multipreconditioned algorithms).
- Apply the new algorithms on large scale problems (*e.g.,* coming from structural mechanics).

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