Topic Proposal for Thematic FMJH/LMH PostDoc – Call 2027

# Unfitted DG solvers for wave propagation in complex geometries

**Keywords:** Finite elements, Numerical analysis, Scientific computing, Wave equation, Helmholtz equation, Fast solvers

**Host laboratory:** Unité de Mathématiques Appliquées (ENSTA) at Palaiseau

**Associated laboratories:** CERMICS (ENPC) and RAPSODI (Inria Lille)

**Contacts:**

- Alexandre Ern, ENPC Professor, CERMICS (alexandre.ern@enpc.fr)
- Théophile Chaumont-Frelet, Inria Researcher, RAPSODI (theophile.chaumont@inria.fr)
- Axel Modave, CNRS Researcher, UMA/POEMS (axel.modave@ensta.fr) *(main contact)*

*Please get in touch before applying so that we can support your application.*

## Scientific context

**Finite element methods** make it possible to solve, with high accuracy, problems formulated as partial differential equations, in particular wave propagation problems of any kind. However, these methods turn out to be **very costly** when dealing with complex three-dimensional geometries in application contexts. A particularly limiting aspect is the need for refined unstructured meshes to describe the interfaces between the different components of the problem under consideration. For time-harmonic waves, refining the mesh increases the size and the solution cost of the linear systems obtained after discretization. In the time domain, it also constrains the time step.

The central idea of this project is to explore, analyze, and numerically test **new unfitted methods** that can be deployed on **simple structured meshes** which do not exactly describe the physical interface, *without sacrificing the accuracy* of the numerical result, while potentially enabling a *faster solution* than classical approaches.

For elliptic or parabolic problems, unfitted methods rely on solid mathematical foundations and have produced very convincing numerical results [1]. For hyperbolic problems, such as those arising in time-dependent wave propagation phenomena, an approach has been proposed for the acoustic system [2] and a formulation based on the second-order wave equation has been analyzed [3]. However, to the best of our knowledge, there is no mathematical analysis providing a rigorous conceptual foundation for *unfitted* methods in **first-order formulation**. Yet this formulation of the wave equation is **fundamental**, since it is the primitive form of the equations. Moreover, highly efficient solution strategies based on discontinuous finite element methods rely on first-order formulations, both for time-dependent waves and for time-harmonic problems.

## Objectives

The first objective of this project is to **develop a mathematical framework** for formulating unfitted discontinuous Galerkin methods to approximate wave propagation in first-order formulations. These formulations fall within the framework of Friedrichs systems [4]. Due to the weak compactness offered by Friedrichs systems, which is much weaker than in the presence of a Laplacian as in the second-order formulation, this question will require subtle numerical analysis.

The second objective is to **propose strategies and analyze possible parameters** to **accelerate the solution** in order to compete with existing solvers. The definition of the scheme and its parameters influence performance. For time-dependent problems, this includes the time step. For time-harmonic ones, it includes substructuring and preconditioning strategies. Since these methods are non-standard, tailored strategies must be developed and studied.

## Proposed Research Directions

Several research directions are proposed for time-harmonic and time-dependent problems.

For time-harmonic problems, we propose carrying out a **frequency-explicit analysis**, following recent advances obtained in particular for traditional finite element methods [5, 6]. The aim will be to extend these results to discontinuous Galerkin methods and to take into account the additional stabilization terms induced by the *unfitted* formulation. Efficient solution strategies for these problems can be investigated, in particular building on the idea of using a **substructuring strategy** based on transmission variables [7].

For time-dependent solvers with implicit time-stepping schemes, mathematical results must be obtained to derive **realistic error bounds** on the numerical approximation and to maintain attractive numerical performance. For explicit schemes, an important new question will be the study of the **CFL stability condition**, and in particular its sensitivity to an *unfitted* formulation. We notably expect to have to add a stabilization term to the mass matrix, and its mathematical study is central to determining the CFL condition. Another important aspect will be the question of **long-time simulations**. Two ideas we wish to explore are, on the one hand, the fact that discontinuous Galerkin methods preserve the involutions intrinsic to the first-order formulation [8], and, on the other hand, the use of the Laplace transform to study the long-time behavior [9].

## Candidate profile

This topic requires expertise in numerical analysis and scientific programming for PDE problems and finite element methods.

## Collaborations

This project is part of a collaboration between Alexandre Ern, Théophile Chaumont-Frelet and Axel Modave. The postdoctoral researcher will be hosted ENSTA in Palaiseau, but visits at ENPC and Inria Lille will be planned.

## References


[1] E. Burman, S. Claus, P. Hansbo, M. G. Larson, and A. Massing. CutFEM: Discretizing geometry and partial differential equations. *Internat. J. Numer. Methods Engrg.*, 104(7), pp. 472–501, 2015. doi:10.1002/nme.4823

[2] S. Schoeder, S. Sticko, G. Kreiss, and M. Kronbichler. High-order cut discontinuous Galerkin methods with local time stepping for acoustics. *Internat. J. Numer. Methods Engrg.*, 121(13), pp. 2979–3003, 2020. doi:10.1002/nme.6343

[3] S. Sticko and G. Kreiss. Higher order cut finite elements for the wave equation. *J. Sci. Comput.*, 80(3), pp. 1867–1887, 2019. doi:10.1007/s10915-019-01004-2

[4] A. Ern and J.-L. Guermond. Discontinuous Galerkin Methods for Friedrichs' Systems. I. General theory. *SIAM J. Numer. Anal.*, 44(2), pp. 753–778, 2006. doi:10.1137/050624133

[5] J. Galkowski and E. A. Spence. Sharp preasymptotic error bounds for the Helmholtz $h$-FEM. *SIAM J. Numer. Anal.*, 63(1), pp. 1–22, 2025. doi:10.1137/23M1546178

[6] T. Chaumont-Frelet, J. Galkowski, and E. A. Spence. Sharp error bounds for edge-element discretisations of the high-frequency Maxwell equations. *Math. Comp.*, 2026. doi:10.1090/mcom/4198

[7] S. Pescuma, G. Gabard, T. Chaumont-Frelet, and A. Modave. A hybridizable discontinuous Galerkin method with transmission variables for time-harmonic acoustic problems in heterogeneous media. *J. Comput. Phys.*, 534, Paper No. 114009, 2025. doi:10.1016/j.jcp.2025.114009

[8] A. Ern and J.-L. Guermond. The Discontinuous Galerkin Approximation of the Grad-Div and Curl-Curl Operators in First-Order Form Is Involution-Preserving and Spectrally Correct. *SIAM J. Numer. Anal.*, 61(6), pp. 2940–2966, 2023. doi:10.1137/23M1555235

[9] T. Chaumont-Frelet and A. Ern. Damped energy-norm *a posteriori* error estimates using $C^2$-reconstructions for the fully discrete wave equation with the leapfrog scheme. *ESAIM Math. Model. Numer. Anal.*, 59(4), pp. 1937–1972, 2025. doi:10.1051/m2an/2025027


*Version: 12/10/2026*